Variation Problem in Elastic Theory of Biomembranes, Smectic-A Liquid Crystals, and Carbon Nanotubes

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The equilibrium shape equation of biomembrane vesicles, a bilayer system, and its solution are studied with Helfrich curvature elastic model, viewed as a surface variation problem in differential geomethry. The biconcave disc shape of red blood cell is obtained analyticly. The torus solutions of membranes are predicted and confirmed by several Labs. The formation of carbon nanotubes is studied as the result with the similar elastic model as biomembranes by which the helical carbon nanotubes are well predicted as those observed in experiment. Equilibrium shapes of domains in Smectic-A liquid crystals, a typical multibilayer system (MBS), have been investigated theoretically. The surface-integral equation and stability condition are obtained by minimizing the free energy of the MBS with their thickness. A surface differential equation is derived from the variation of the energy. The latter concerns the most general differential equation of surface which is the Euler-Lagrange equation for the variation problem $\delta \oint \Phi(H,K)dA = 0$. Here Φ is any function of the mean curvature H and Gaussian curvature K of the surface, i. e. the generalized Helfrich curvature free energy.

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