The Dirac operator was created by the physicist Paul Adrien Maurice Dirac about 1928 to give a description of electrons combining both quantum and (special) relativistic approaches to the physical reality. However, the basic objects involved in the definition of this operator, namely the spinor fields, had been already introduced by Élie Cartan in 1913 when he studied orthogonal representations of Lie algebras and reinvented by Wolgang Pauli in 1927 who used them to model the inner angular momentum (spin) of electrons. In the thirties, Richard Brauer, Hermann Weyl and Élie Cartan again developed the mathematical foundations relative to these spinor fields and the Dirac operator, but, in 1937, in the introduction of his last book, *La théorie des spineurs*, Cartan asserted that there were "insurmountable difficulties" to talk about these mathematical objects on non-flat Riemannian or Lorentzian manifolds. Yet these difficulties were overcome by Michael Atiyah and Isaac Singer in 1963. They showed that, in fact, it is possible to have spinor fields and Dirac operators on all the orientable Riemannian manifolds with vanishing second Stiefel-Whitney class. That is, the "insurmountable difficulties" were reduced to a small topological obstruction. Atiyah and Singer turned the Dirac operator into a mathematical cornerstone by showing their celebrated Index Theorem. After that, Lichnerowicz, in the sixties, and Gromov and Lawson, in the eighties, saw that this operator played a fundamental role in the interaction between geometry and (scalar) curvature on Riemannian manifolds. The last two surprising applications of this Dirac operator are due to Edward Witten. He found in 1981 an *elementary* spinorial proof of the positive energy theorem by Schoen and Yau and in 1995 proposed the study of the so-called Seiberg-Witten equations to obtain in a new and easier way the Donaldson results about topology of four-manifolds.
On the other hand, from the physical point of view, the Dirac operator has remained to be, from its birth itself, the main tool to describe the fermionic particles and so plays a central role in the modern *supersymmetric* theories, where tensor (bosonic) and spinor (fermionic) fields are interchanged.

In this mini-course we will intend to provide elements for an elementary and quick introduction to the geometry and the analysis of the Dirac operator on Riemannian manifolds. We will try to avoid an excessive algebraic or topological machinery and we will focus on the geometrical aspects of this operator.