

Harmonic Spheres in Loop Spaces of Compact Lie Groups

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Atiyah has established a 1-1 correspondence between holomorphic spheres in the loop space ΩG of a compact Lie group G and G -instantons on \mathbb{R}^4 . Motivated by this result, we conjecture that Yang-Mills G -fields on \mathbb{R}^4 should correspond to harmonic spheres in ΩG .

To describe harmonic spheres in ΩG , we embed the loop space ΩG into an infinite-dimensional Grassmanian $Gr(H)$ of a Hilbert space H . For the description of harmonic spheres in this Grassmanian, we use the twistor approach, which reduces the real problem of constructing harmonic spheres in $Gr(H)$ into a complex problem of constructing holomorphic spheres in a twistor bundle of $Gr(H)$, given by a certain virtual flag bundle over $Gr(H)$.