Group Analysis as a Microscope of Mathematical Modelling

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Abstract

Most of mathematical models in physics, engineering sciences, biomathematics, etc. are formulated as systems of nonlinear differential equations. Accordingly, engineering and science students and researchers routinely confront problems in mathematical modeling involving solution techniques for differential equations. Sometimes these solutions can be obtained analytically by numerous traditional ad hoc methods appropriate for integrating particular types of equations. More often, however, the solutions cannot be obtained by these methods, in spite of the fact that over 400 types of integrable second-order ordinary differential equations were summarized in voluminous catalogues.

On the other hand, the fundamental natural laws and technological problems formulated in terms of differential equations can be successfully treated and solved by Lie group methods. For example, Lie group analysis reduces the classical 400 types of equations to 4 types only! Development of group analysis furnished ample evidence that the theory provides a universal tool for tackling considerable numbers of differential equations even when other means of integration fail. In fact, group analysis is the only universal and effective method for solving nonlinear differential equations analytically. The old integration methods rely essentially on linearity as well as on constant coefficients. Group analysis deals equally easily with *linear and nonlinear* equations, as well as with constant and variable coefficients.

Lie group analysis can be called a microscope of mathematical modelling. Just like use of microscope in biology, group analysis allows to reveal symmetries of nonlinear mathematical models that cannot be seen otherwise. Numerous physical phenomena can be investigated using Lie symmetries to unearth various group invariant solutions and conservation laws that provide significant physical insight into the problem. An extensive compilation of the results of applications of Lie group analysis up to 1995 is given in three volumes [7].

The aim of this lecture course is to provide the wide audience of researchers and students with a comprehensive introduction to modern group analysis and to illustrate the advantages of Lie group analysis by various examples.

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