# Systems of MKdV equations related to the affine Lie algebras

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We have derived a family of mKdV-type equations related to the affine Lie algebras $\mathfrak{g}$ using a Coxeter $\mathbb{Z}_h$-reduction where $h$ is the Coxeter number of $\mathfrak{g}$. Each of these systems of equations is Hamiltonian. For the algebra $sl(r+1)$ it reads:

$$\frac{\partial q_i}{\partial t} = \frac{\partial}{\partial x}\left( \frac{\delta H}{\delta q_{r+1-i}} \right). \tag{1}$$

In particular for $\mathfrak{g} \simeq sl(5)$, i.e. $r = 4$ we obtain:

$$\begin{aligned} H = \frac{2b}{3a^3} \int_{-\infty}^{\infty} dx \Bigg( & -\frac{c_1}{2s_1^2} \frac{\partial q_1}{\partial x} \frac{\partial q_4}{\partial x} + \frac{c_2}{2s_2^2} \frac{\partial q_2}{\partial x} \frac{\partial q_3}{\partial x} + q_1 q_3^3 + q_2^3 q_4 + q_3 q_4^3 \\ & + \frac{3}{8s_1} \left( q_4^2 \frac{\partial q_2}{\partial x} - 2q_2 q_4 \frac{\partial q_4}{\partial x} + 2q_1 q_3 \frac{\partial q_1}{\partial x} - q_1^2 \frac{\partial q_3}{\partial x} \right) + 3q_1 q_2 q_3 q_4 + q_1^3 q_2 \\ & + \frac{3}{8s_2} \left( q_2^2 \frac{\partial q_1}{\partial x} - 2q_1 q_2 \frac{\partial q_2}{\partial x} + 2q_3 q_4 \frac{\partial q_3}{\partial x} - q_3^2 \frac{\partial q_4}{\partial x} \right) \Bigg). \end{aligned} \tag{2}$$

where $s_k = \sin(k\pi/5)$ and $c_k = \cos(k\pi/5)$, $k = 1, 2$, i.e.

$$s_{1,2} = \frac{1}{4}\sqrt{10 \mp 2\sqrt{5}}, \qquad c_{1,2} = \frac{1}{4}(1 \pm \sqrt{5}). \tag{3}$$

We also derive the recursion operators and demonstrate their Hamiltonian hierarchies. Similar results can be derived also for affine algebras of higher rank.

## References

[1] V. S. Gerdjikov, D. M. Mladenov, A. A. Stefanov, S. K. Varbev. Integrable equations and recursion operators related to the affine Lie algebras $A_r^{(1)}$. **ArXiv: 1411.0273v1 [nlin-SI]** JMP (In press)