Toroidal Surfaces

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Abstract

We show that the 2-torus in $\mathbb{R}^3$ is a critical point of a sequence of functionals $\mathcal{F}_n$ $(n = 1, 2, 3, \cdots)$ defined over compact 2-surfaces (closed membranes) in $\mathbb{R}^3$. When the Lagrange function $\mathcal{E}$ is a polynomial of degree n of the mean curvature H of the surface, the radii (a, r) of the 2-torus are related as $\frac{a^2}{r^2} = \frac{n^2-n}{n^2-n-1}$, $n \geq 2$. A simple generalization of 2-torus in $\mathbb{R}^3$ is a tube of radius r along a curve α which we call it toroidal surface (TS). We show that toroidal surfaces with non-circular curve α do not provide minimal energy surfaces of the functionals $\mathcal{F}_n$ $(n = 2, 3)$ on closed surfaces. We discuss possible applications of the functionals discussed in this work on cell membranes.