Painlevé Test and the Resolution of Singularities for Integrable Equations

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ABSTRACT

The Painleve property has been regarded as one major indicator of the integrability of differential equations. Kovalevskaya used the property in her discovery of all the integrable cases of the spinning top. Painleve used the property in his discovery of his six transcendents. The Painleve test is the formal process inspired by the property and has been a very useful tool for detecting the integrability of differential equations. For example, the test played a central role in Adler and Moerbecks theory of complete algebraic integrability.

Despite the usefulness of the Painleve test, the test has long been a formal algebraic process, and the basic question of the analytic meaning of the test remains unanswered. We discovered that, in the simplest case, passing the Painleve test is equivalent to the existence of an explicit change of variable that can be considered as resolving the movable singularities. This suggests that, while the algebraic process of the Painleve test is comparable to the formal power series solutions, the test also has the analytic meaning similar to the Cauchy-Kovalevsky theorem.