Symmetry, Phases and Quantisation

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We start with the basic concepts in classical mechanics, exploring the consequences of symmetries in Lagrangian and Hamiltonian systems. We then use the linear symplectic system as an example to illustrate the procedure of quantisation and the rich structures that emerges. This leads naturally to geometric quantisation of arbitrary symplectic manifolds, as the concepts involved already appeared in the linear systems. We also comment on quantisation in the presence of symmetries. We then compare other forms of quantisation, such as deformation quantisation, path integral quantisation and quantisation via branes. Finally, we study geometric phases in classical and quantum mechanics. The detailed contents of the lectures can be seen below.

Lecture 1: Symmetries in Lagrangian and Hamiltonian mechanics. Lagrangian mechanics, Noether's theorem, Legendre transform, Hamiltonian mechanics, moment maps, symplectic quotients, examples.
Lecture 2: Quantisation of linear systems. Quantisation of linear symplectic systems, Fourier and Segal-Bargmann transforms, projectively flat connections, Maslov index, quantisation of fermionic systems.
Lecture 3: Outline of geometric quantisation. Prequantisation, polarisation, half-density and half-forms, metaplectic correction, inner product and BKS pairing, change of polarisations, quantisation and reduction.
Lecture 4: Relation to other forms of quantisation. Deformation quantisation, path integral quantisation, quantisation via branes.
Lecture 5: Geometric phases in classical and quantum systems. Hannay's angle, Berry's phase, classical and quantum adiabatic theorems, geometric phases in the presence of symmetries, relation to the holonomy of a projectively flat connection.

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