On Auto and Hetero Backlund Transformation for the Hamilton-Jacobi Equations

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On auto and hetero Backlund transformation for the Hamilton-Jacobi equations The primary aim of the study was to investigate Backlund transformations (BT) and to discuss some correspondence between the integrable cases of the Henon-Heiles system. This correspondence may be considered a finite-dimensional counterpart of the hetero Backlund transformation that can be used for building and classifying of integrable systems.

An application of the possible finite-dimensional analogues of the well-studied Backlund transformations for nonlinear differential equations to three Henon-Heiles systems is discussed. Using the known 2x2 Lax matrix and special similarity transformation we obtain new unique Lax matrix with the following properties:

- first off-diagonal element of Lax matrix defines initial parabolic coordinates on the plane $v_1, 2$;
- second off-diagonal element has only two commuting and functionally independent zeroes $u_1, 2$;
- the conjugated momenta for u and v variables are the values of the diagonal element;

Thus we have two families of variables of separation for the first Henon-Heiles system and can explicitly define canonical transformation between them.

After that we converted this special auto-BT to some analogue of the hetero-BT which relates different Hamilton-Jacobi equations associated with the three different integrable cases of the Henon-Heiles system.