On a Method for Solving of Multidimensional Equations of Mathematical Physics

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We consider linear multidimensional evolutionary equations or the linear part of nonlinear ones. The complex structure and the presence of terms with different physical sense requires the coordinate splitting to be preceded by splitting by physical factors (processes). In contrast to the coordinate splitting this kind of splitting can be exact in some nodes and in the intervals it can be controlled. The method in question is developed in 70s of 20th century by G. Marchuk and it is applied very successfully for solving of various problems in ecology, air and water pollution, diffusion, etc.

In this paper we aim to demonstrate that the splitting by physical factors is applicable and can be efficient for solving of multidimensional evolutionary problems in the optics. Without loosing of generality we pay attention to an initial-value problem of (3+1)D equations of Schrodinger kind. We split the linear part of the differential operator and the initial conditions to two consequent Cauchy problems and by using a spectral analysis and techniques we proof that the original and the resulting splitting problems are equivalent in the ends of a given interval. For constant coefficients the above assertion holds in the whole interval. In opposite case for small enough step the approximation error varies in an acceptable range.

The method is applied successfully for numerical solving of (3+1)d Schrodinger equation with sign-variable group velocity as well for (3+1)d amplitude equations (of envelope), when the propagation regimes of the light pulses may be ultra-short (femtosecond). The obtained results are reliable and give good predictions for the material quantities and dynamics of the light pulses.