On Charge Conservation in a Gravitational Field

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ABSTRACT

The Maxwell equations of special relativity imply the charge conservation, as is well known. The same is true for their standard extension to a curved spacetime, which is obtained by replacing the partial derivatives with the covariant derivatives associated with the metric. This replacement contains implicitly the assumption of Einstein dynamics in a curved spacetime, according to which free test particles follow geodesic lines of the spacetime metric. However, an alternative dynamics can be defined from an extension to curved spacetime of the special-relativistic form of Newton's second law. In that extension the gravity acceleration is assumed to derive from a spatial potential, as in Newtonian theory, and geodesic motion is got merely for a static metric [1]. It implies a preferred reference frame. This dynamics can be extended to a continuous medium or to a system of fields, also in the presence of an external force field. When the latter is the Lorentz force due to an electromagnetic field, that dynamics implies, under natural assumptions, a definite form for the Maxwell equations in a gravitational field [2]. That form does not imply the exact charge conservation.

The aim of the present work is to evaluate the amounts of the (macroscopic) charge production or destruction which are thus predicted, in physically relevant situations. This task is not easy because the modified second group of the Maxwell equations can be written in simple form only if the electric and magnetic fields are not orthogonal. We introduce an asymptotic expansion scheme for the electromagnetic field in a weak gravitational field. As a consequence of this scheme, the first approximation of the gravitationally-modified Maxwell equations corresponding with the alternative dynamics coincides with the Maxwell equations of special relativity, as expected. We show that, for a plane wave, there is no charge production or destruction if one neglects a term involving $\partial_T(\nabla U)$ where U is the Newtonian potential — which term is usually extremely small. We show however that, for a group of Hertzian dipoles, the charge production/destruction takes unrealistically high values. Thus this version of the gravitationally-modified Maxwell equations has to be discarded. We find that the "natural" assumptions made to derive them imply in fact a hidden inconsistency with another equation of the theory. We present consistent equations which are in agreement with the theory and predict charge conservation.

References

[1] M. Arminjon, On the extension of Newton's second law to theories of gravitation in curved space-time, *Arch. Mech.* **48**, 551-576 (1996).

[2] M. Arminjon, Continuum dynamics and the electromagnetic field in the scalar ether theory of gravitation, *Open Physics* **14**, 395-409 (2016).