Individual Ergodic Theorems in Semifinite von Neumann Algebras

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ABSTRACT

The aim of this talk is to present some recent developments in the noncommutative ergodic theory with a positive Dunford-Schwartz operator acting in a semifinite von Neumann algebra (M, τ). These developments are, to a great degree, due to the following: A refinement of the noncommutative Banach Principle for the Banach space $L^p(M, \tau)$,$1 \leq p < \infty$, where the bilaterally uniform boundedness of a sequence of positive linear continuous maps $A_n : L^p(M, \tau) \to L^0(M, \tau)$(the algebra of τ-measurable operators) implies that the set of all such $x \in L^p(M, \tau)$ that the sequence $\{A_n(x)\}$ converges almost uniformly is closed in $L^p(M, \tau)$. This allowed to establish almost uniform convergence for a variety of classes of ergodic averages acting in the noncommutative space $L^p(M, \tau)$, which for a long time had been known to converge only bilaterally almost uniformly. It was shown that if $\mu_t(x)$ is a non-increasing rearrangement of a τ-measurable operator x, then the fully symmetric space

$$R_\mu = \{x \in L^1(M, \tau) + L^\infty(M, \tau) : \mu_t(x) \to 0 \ \text{ as } \ t \to \infty\}$$

is, in a sense, the largest subspace of $L^1(M, \tau) + L^\infty(M, \tau)$ in which the almost uniform convergence of noncommutative ergodic averages holds.