On Effective Classification of PDEs with Algebraic Coefficients

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ABSTRACT

On effective classification of PDEs with algebraic coefficients Abstract: The problem of studying the action of contact or point transformations on various classes of differential equations is one of the most important problems in mathematics and has been studied by many prominent mathematicians, including S. Lie, E. Cartan, etc. The main methods in their works are based on ideas of the geometry of differential equations, which has a long history and is extensively studied nowadays.

However, most of results in this area have two significant disadvantages: they are local (i.e. they can be applied only in the neighborhood of some point) and they are not computable (with the help of the computer). On the other hand, during the last years in some works there were developed different applications of the theory of differential equations in algebra and algebraic geometry and vice versa.

In this work we suggest a modification of the classical problem of classification of differential equations, which makes it possible to obtain an effective equivalence criterion for two given differential equations. Namely, we consider not arbitrary equations, but only equations with algebraic coefficients. We use the ideas from theory of differential invariants on the one hand and from algebraic geometry on another hand to provide the effective equivalence criterion for two such equations. In particular, we present the effective classification of second order ODEs y'' = F(x, y, y') (in smooth case these equations were studied by S. Lie, A. Tresse, B. Kruglikov and others). The author is supported by RFBN, grant 16_01_60018 mol_a_dk.