Planar p-Elasticae and Rotational Linear Weingarten Surfaces

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ABSTRACT

A curve immersed in the Euclidean 3-space is called an elastic curve (or elasticae) if it is a critical point of the bending energy. In 1744, L. Euler described all possible qualitative types for planar elasticae.

Here, we consider a family of generalized bending energies acting on planar curves. Critical points of these generalizations are going to be called p- elasticae. For these energies, we derive the Euler-Lagrange equation and its first integral. Then, using a technique which involves the associated Killing vector fields along critical points, we generate binormal evolution surfaces, which have a nice geometric property. In fact, they are rotational linear Weingarten surfaces.

Now, after giving the classification of rotational linear Weingarten surfaces of Euclidean 3-space, we prove that, locally, all of them are foliated by p- elasticae being orthogonal to the rotation. That is, we obtain that a rotational surface is a linear Weingarten surface, if and only if, its generating curve is a planar p-elasticae.

Finally, we particularize our findings to two remarkable cases. Firstly, we consider classic elastic curves, and due to our construction, we recover the characterization of generating curves of Mylar balloons as elastic curves and the well-known parametrization of these balloons in terms of Elliptic functions. Secondly, by considering critical curves of a Blasche's variational problem (which we will prove that they represent the roulettes of conic foci), we obtain all constant mean curvature rotational surfaces.