

Flocking Dynamics on Riemannian Manifolds

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ABSTRACT

The dynamics of the Cucker-Smale model facilitate the flocking of a group of particles in disordered motion into a coordinated one where all particles move parallelly with the center of mass. It illustrates not only the flocking of animals but also the general emergence of collective behaviour in a wide range of subjects. While this work already received much attention over the last decade, those efforts focused on particles moving in a Euclidean space.

We generalise the model to complete Riemannian manifolds and establish theorems about the convergence of the particles to a flocked state. On a Riemannian manifold the notion of parallelism is intimately related to curvature and the geometry constrains the final flocked state into specific patterns. Next to this interesting phenomenology, our work also is a contribution towards the so-called flocking realizability problem. Given a manifold and a group of particles, this problem asks for a dynamical system that leads to a collective movement as a flock at least asymptotically. This project is joint work with Seung-Yeal Ha and Doheon Kim from Seoul National University.