Symmetry Groups of Systems of Entangled Particles

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ABSTRACT

A Lorentz transformation of signature (m, n), $m, n \in \mathbb{N}$, is a pseudo-rotation in a pseudo-Euclidean space of signature (m, n). Accordingly, the Lorentz transformation of signature $(1, 3)$ is the common Lorentz transformation from which special relativity theory stems. It is widely acknowledged that special relativity and quantum theories are at odds. In particular, it is known that entangled particles involve Lorentz symmetry violation. Hence, we pay attention to Lorentz transformations of signature (m, n) for all $m, n \in \mathbb{N}$. It turns out that the Lorentz transformations of signature (m, n) form the symmetry group by which systems of m n-dimensional entangled particles can be understood, just as the common Lorentz group of signature $(1, 3)$ forms the symmetry group by which Einstein's special theory of relativity can be understood. Consequently, it is useful to extend special relativity theory by incorporating Lorentz transformation groups of signature $(m, 3)$ for all $m \geq 2$. The resulting extended special relativity theory, then, provides not only the symmetry group of the $(1+3)$-dimensional spacetime of particles, but also the symmetry group of the $(m+3)$-dimensional spacetime of systems of m entangled 3-dimensional particles, for each $m \geq 2$. A systems of m entangled particles has 3 spatial dimensions since the m particles share the 3-dimensional world where we live. However, the system has m temporal dimensions since each of the m particles has its own time measured by its own internal clock, so that the m-dimensional time of the system is measured by m entangled clocks. A novel, unified parametrization of the Lorentz transformations of signature (m, n), $m, n \in \mathbb{N}$, shakes down the underlying matrix algebra into elegant and transparent results.

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