Application of Geometrical Methods to Study the Systems of Differential Equations for Quantum-Mechanical Problems

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ABSTRACT

In the present work, a geometrical method based on the structural stability theory is used to study systems of differential equations which arise in quantum-mechanical problems. Illustrative examples rely on considering a 1/2- spin particle in external Coulomb field or in the presence of magnetic charge on the background of the de-Sitter space, a free 3/2-spin particle in spherical coordinates of the flat space, or a vector particle in the Coulomb potential. Different situations emerge: in some cases, the system of differential equations can be diagonalized, while in other cases, by using linear transformations, the system of equations can be reduced to a form with a simpler structure of singular points. Moreover, the needed flattening transformations may depend on the argument of the functions. For all the considered systems, it turns out that the first and the second Kosambi-Cartan-Chern invariants are nontrivial, while the 3-d, 4-th and 5-th invariants identically vanish. The first invariant determines the vector field on the configuration space of the differential system, and is interpreted as an external field potential. From physical point of view, the second invariant is the most interesting, since it determines how rapidly the different branches of the solution diverge from or converge to the intersection points, which usually are the singular ones. It is shown that the behavior of the eigenvalues of the second Kosambi-Cartan-Chern invariant is the same for the complicated initial system, and for the transformed simplified one. The vanishing of the 3-d, 4-th and 5-th invariants means that, in geometrical terms, there exists a nonlinear connection on the tangent bundle, with zero torsion and curvature. Additionally, in each case we determine the metric function L underlying the vector field of the differential system, and the metric tensor g_ik of the related geometric space. However, for the case of a vector particle in the Coulomb field, the problem of finding the metric function L is intricate, since it leads to a nontrivial Riccati equation.