The Helfrich Model for the Elasticity of Biomembranes as a Limit of Mesoscopic Energies

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ABSTRACT

Biomembranes are remarkable structures with both fluid-like and solid-like properties: the main constituents are amphiphilic lipids, which have a head part that attracts water and a tail part that repels it. As a consequence, such lipids organise themselves in micelle and bilayer structures, where the head parts shield the lipid tails from the contact with water. In a recent paper by Peletier and Röger (2009), a mesoscale model was introduced in the form of energy for idealised and rescaled head and tails densities. The energy has two contributions; one penalises the proximity of tail to polar (head or water) particles, and the second implements the head-tail connection as an energetic penalisation. The thickness of the structure is minimal, and a full Gamma-convergence result has been proved in the 2D case. The Gamma-limit turns out to be the Euler elastic functional for curves in the plane. The 3D case is much harder, and there are only partial results. In this seminar, I will present the mesoscopic model proposed by Peletier and Röger. I will briefly explain how the deduction of the 2D-macroscopic model by Gamma-convergence works and then he will provide some details on the 3D-case, where we obtain an Helfrich-type energy as a limit. The analysis of such a case requires in-depth tools from geometric measure theory, like currents and varifolds, to have weak notions of surfaces suitable for Calculus of Variations and for which a suitable notion of curvatures exists. This research project is in collaboration with Mark Peletier and Matthias Röger.