- Boundary conditions

$$\mathbf{e}_2 \cdot \nabla \left[\frac{\partial \mathcal{E}}{\partial (2H)} \right] + \mathbf{e}_2 \cdot \tilde{\nabla} \left(\frac{\partial \mathcal{E}}{\partial K} \right) - \frac{d}{ds} \left(\tau_g \frac{\partial \mathcal{E}}{\partial K} \right) + \frac{d^2}{ds^2} \left(\frac{\partial \Gamma}{\partial k_n} \right) + \frac{\partial \Gamma}{\partial k_n} (k_n^2 - \tau_g^2)$$
$$+ \tau_g \frac{d}{ds} \left(\frac{\partial \Gamma}{\partial k_g} \right) + \frac{d}{ds} \left(\tau_g \frac{\partial \Gamma}{\partial k_g} \right) - \left(\Gamma - \frac{\partial \Gamma}{\partial k_g} k_g \right) k_n \Big|_C = 0,$$

$$-\frac{\partial \mathcal{E}}{\partial (2H)} - k_n \frac{\partial \mathcal{E}}{\partial K} + \frac{\partial \Gamma}{\partial k_g} k_n - \frac{\partial \Gamma}{\partial k_n} k_g \Big|_C = 0,$$

$$\frac{d^2}{ds^2} \left(\frac{\partial \Gamma}{\partial k_g} \right) + K \frac{\partial \Gamma}{\partial k_g} - k_g \left(\Gamma - \frac{\partial \Gamma}{\partial k_g} k_g \right) + 2(k_n - H) k_g \frac{\partial \Gamma}{\partial k_n}$$

$$-\tau_g \frac{d}{ds} \left(\frac{\partial \Gamma}{\partial k_n} \right) - \frac{d}{ds} \left(\tau_g \frac{\partial \Gamma}{\partial k_n} \right) - \mathcal{E} \Big|_C = 0.$$

Morphological problems of lipid bilayers

Lipid vesicles

$$\Delta p - 2\lambda H + k_c \nabla^2 (2H) + k_c (2H + c_0)(2H^2 - c_0 H - 2K) = 0$$

- Sphere $\Delta p R^2 + 2\lambda R - k_c c_0 (2 - c_0 R) = 0$

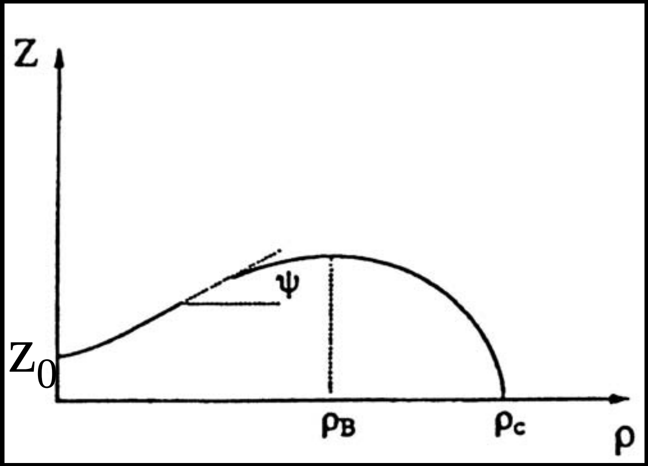
1 root: If $(2\lambda + k_c c_0^2)^2 + 8\Delta p k_c c_0 = 0$

2 roots: If $(2\lambda + k_c c_0^2)^2 + 8\Delta p k_c c_0 > 0$

- Red blood cell---biconcave shape

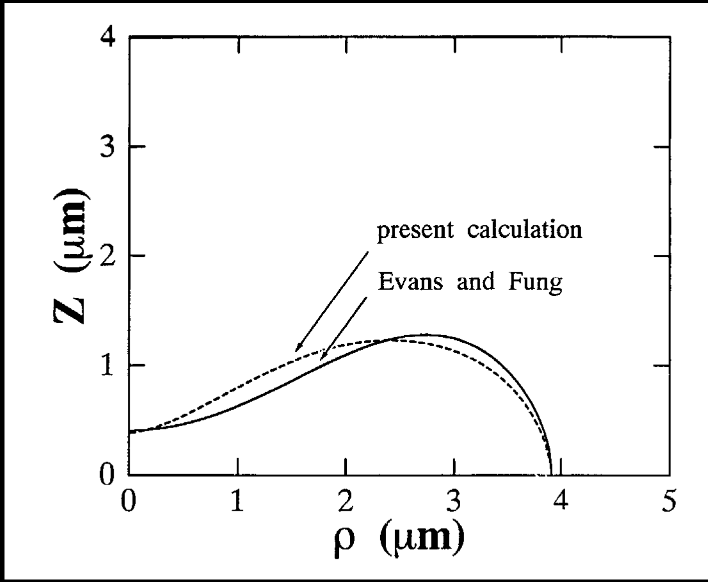
$$z = z_0 + \int_0^\rho \tan\psi(\rho')d\rho'$$

$$\sin\psi(\rho) = c_0\rho\ln(\rho/\rho_B), c_0 < 0$$

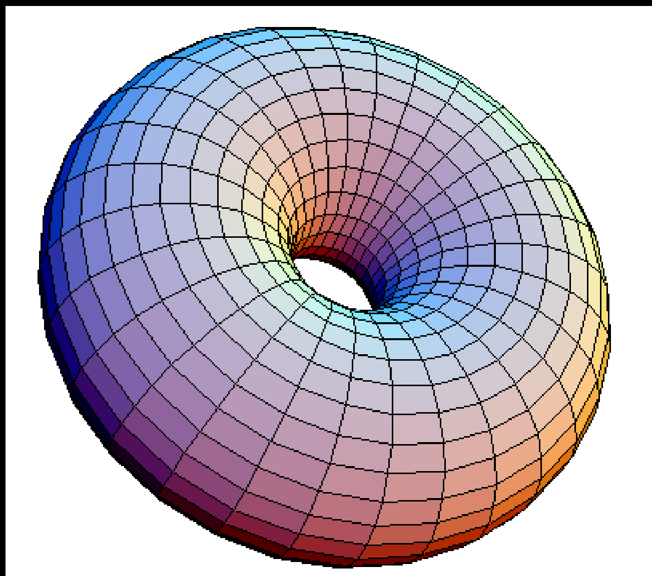


[Ou-Yang, Hu J.G., & Liu J.X. 1992]

[Evans & Fung, Microvasc. Res. **4** (1972) 335]

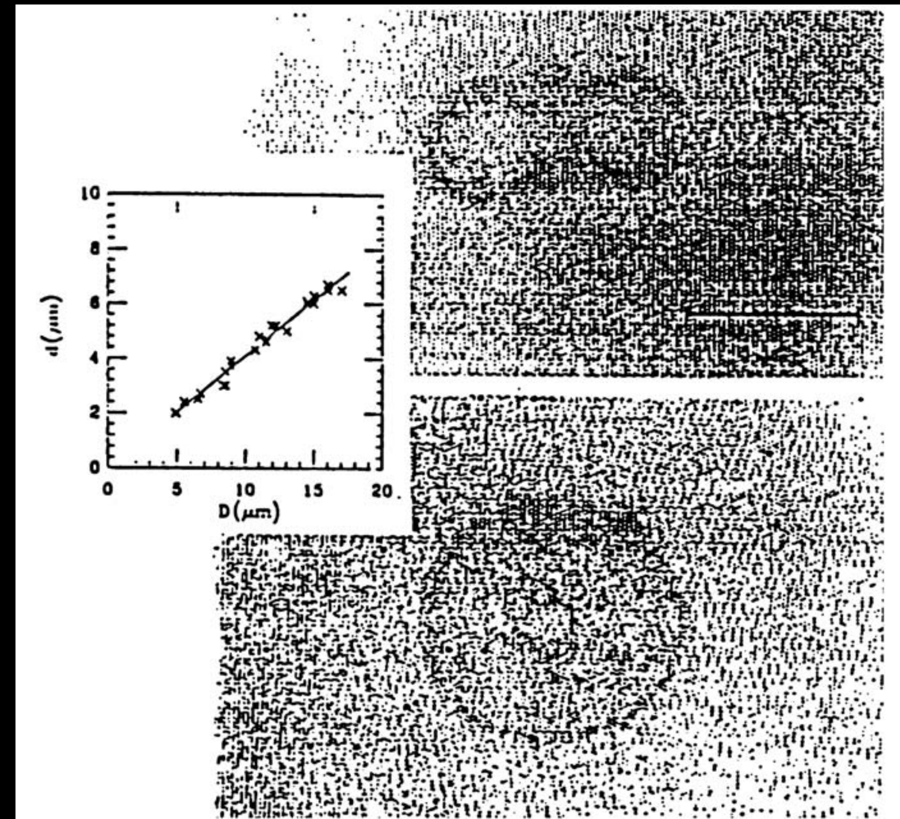


- Torus



$$R / r = \sqrt{2}$$

[Ou-Yang 1990 PRA **41** 4517]



Confirmed by experiments:

- M. Muty & D. Bensimon, PRA, 1991, 24 tori;
- A.S. Rudolph et al, Nature, 1991, in Phospholipid membrane;
- Z. Lin et al, Langmuir, 1994, in Micelles.

Open lipid bilayers

$$k_c(2H+c_0)(2H^2-c_0H-2K)-2\lambda H+k_c\nabla^2(2H)=0$$

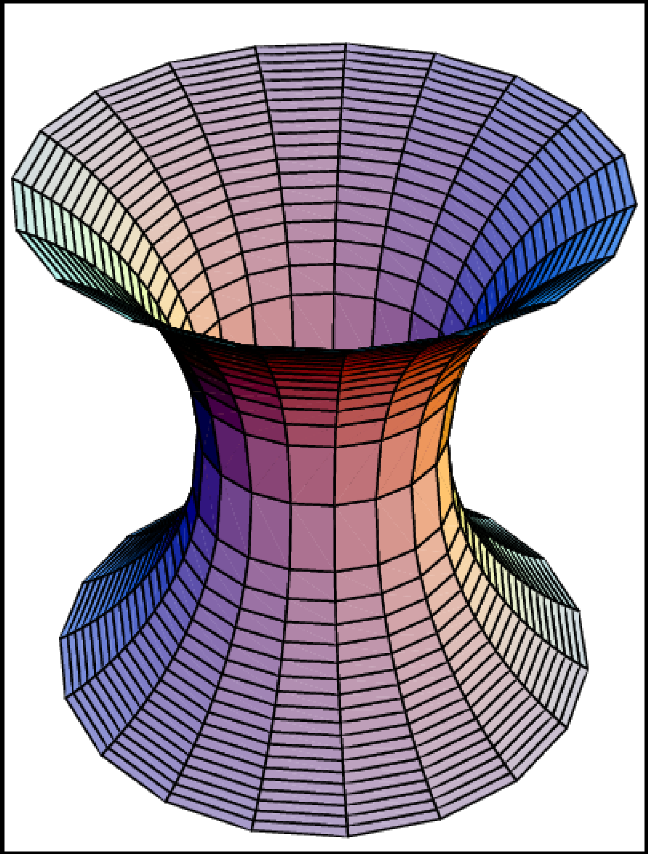
$$\left[k_c(2H+c_0)+\bar{k}k_n\right]\Big|_C=0$$

$$\left[-2k_c\frac{\partial H}{\partial \mathbf{e}_2}+\gamma k_n+\bar{k}\frac{d\tau_g}{ds}\right]\Bigg|_C=0$$

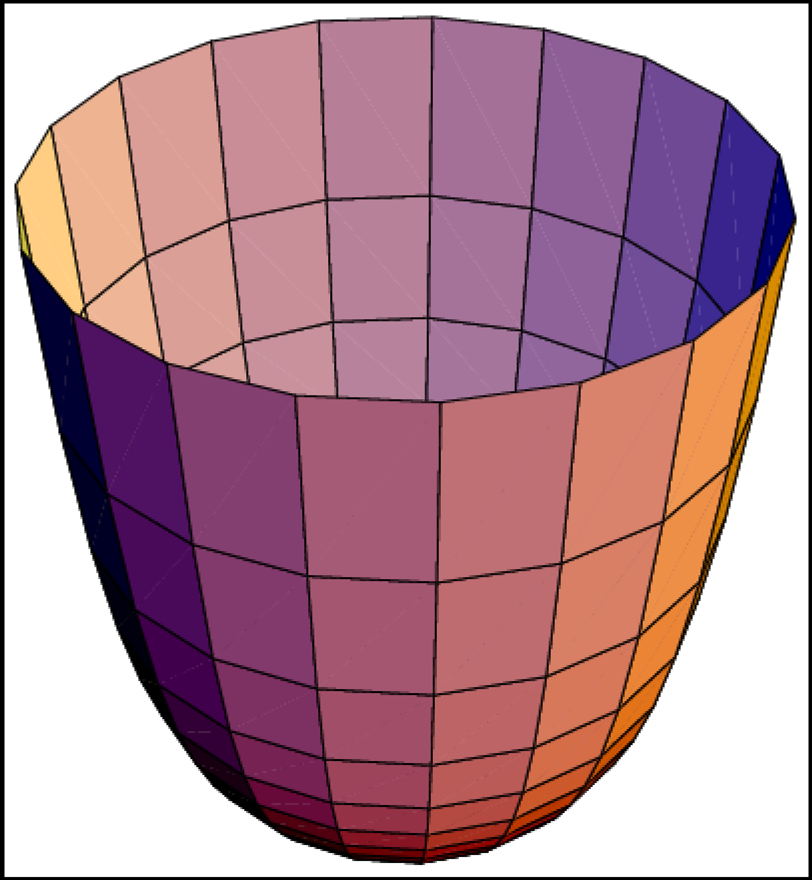
$$\left[\frac{k_c}{2}(2H+c_0)^2+\bar{k}K+\lambda+\gamma k_g\right]\Bigg|_C=0$$

- No axisymmetric constant mean curvature surface with edges

- Central part of the torus



- Cuplike membrane



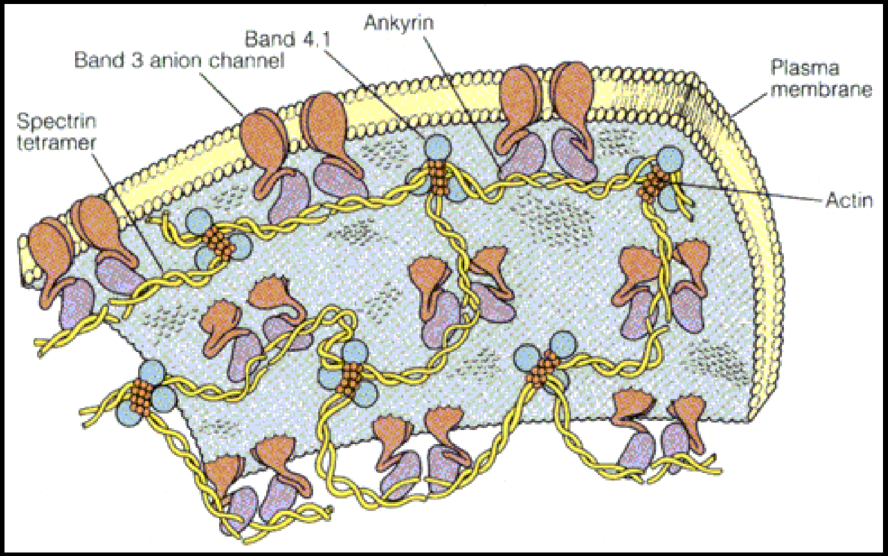
[Tu & Ou-Yang PRE **68** (2003) 61915]

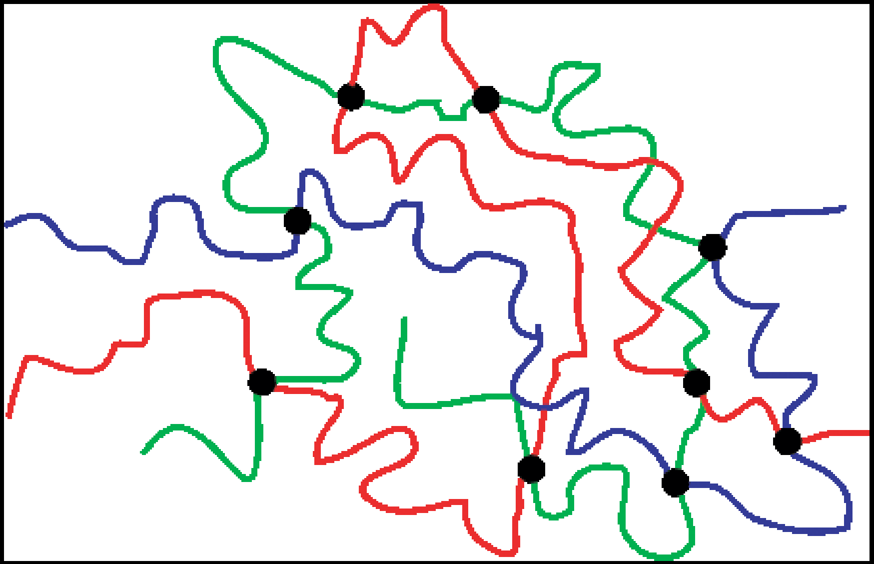
Elasticity and stability of cell membranes

[JPA **37** (2004) 11407]

Simplified model

- Cell membrane=lipid bilayer+membrane skeleton
- Membrane skeleton = cross-linking protein network = rubber membrane





[Schematic structure of MSK (left) and rubber (right) at molecular levels]

Free energy

$$\mathcal{F} = \int_M (\mathcal{E}_d + \mathcal{E}_H)\, dA + p \int_V dV$$

$$\mathcal{E}_H = (k_c/2)(2H + c_0)^2 + \lambda$$

$$\mathcal{E}_d = (k_d/2)[(2J)^2 - Q]$$

$$2J = \varepsilon_{11} + \varepsilon_{22},\ Q = \varepsilon_{11}\varepsilon_{22} - \varepsilon_{12}^2$$

Small deformation; remain to the second order terms