0-Brane Matrix Dynamics for QCD purposes: Regge Trajectories

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Basic Idea

Matrix Dynamics

Quantum Dynamics

Angular momentum

Harmonic osc.

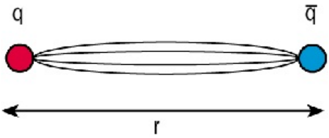
Rayleigh-Ritz Method and Spectrum

High energy Hadron-Hadron scatterings show two regimes:

1) Large momentum transfer: interaction among point-like substructures

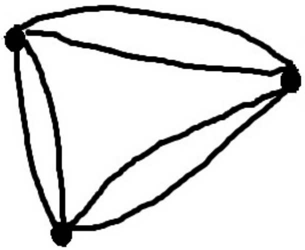
2) Small momentum transfer: linear Regge trajectories are exchanged $\Rightarrow$ Motivation for String picture

Possible reconciliation of two regimes: Hadrons as bound-states of point-like Quarks and QCD flux-tubes (QCD-Strings).



Field theory anomalies of 2-dim string world-sheet $\Rightarrow$ Lack of consistent QCD-Strings in 3+1 dims.

0-branes: Point-like objects to which strings end



Coordinates of N 0-branes given by $N \times N$ hermitian matrices $\Rightarrow$ Strings' dynamics encoded in off-diagonal elements of matrices

Suggestion: Modelling bound-state of Quarks and QCD-Strings by 0-brane matrix dynamics $\Rightarrow$ Encouraging features [Fatollahi, EPL **53**, **56**, EPJC **19**, **27**, **17**]:

1) Linear potential between static 0-branes

2) Regge behavior in scattering amplitudes

3) Whiteness of 0-branes' c.m. w.r.t U(1) gauge fields on matrix space

Q: Do linear Regge trajectories emerge from

0-brane matrix dyn.? $\Rightarrow$

Check Energy spectrum vs. Ang. Mom.

Q: Advantage of QM to world-sheet theory?

A: Absence of anomalies in QM of finite matrices

$\Rightarrow$ Possible consistent theory in 3+1 dims.

Q: Relevance of Matrix Coordinates to QCD physics?

A: 1) Special relativity lesson: 4-vector photon fields live in 4-vector space-time coordinates

2) SUSY lesson: anti-commuting coordinates represent fermion content

Matrix[YM] Fields $\Leftrightarrow$ **Matrix Coordinates** $\rightarrow$ **Who knows about the exact nature of Coordinates inside a proton?**

Dynamics of N 0-branes given by U(N) YM theory

dimensionally reduced to $0+1$ dimensions:

$$L = m_0 \mathrm{Tr}\left(\frac{1}{2}(D_t X_i)^2 + \frac{1}{4\, l_s^2}\,[X_i, X_j]^2\right)$$

$$i, j = 1, ..., d, \qquad D_t = \partial_t - \mathrm{i}[A_0,\ ]$$

X: $N \times N$ hermitian matrices, $\quad X_i = x_{i\,a} T_a$

l_s: string length, $\quad m_0 = (g_s l_s)^{-1}$

$m_0 \gg l_s^{-1}$.

Theory is invariant under the gauge symmetry

$$\begin{aligned} \vec{X} &\rightarrow \vec{X}' = U\vec{X}U^\dagger, \\ A_0 &\rightarrow A_0' = UA_0U^\dagger + \mathrm{i}U\partial_t U^\dagger, \end{aligned} \tag{1}$$

U: arbitrary time-dependent $N \times N$ unitary matrix

$$\begin{aligned} D_t\vec{X} &\rightarrow D_t'\vec{X}' = U(D_t\vec{X})U^\dagger, \\ D_tD_t\vec{X} &\rightarrow D_t'D_t'\vec{X}' = U(D_tD_t\vec{X})U^\dagger. \end{aligned} \tag{2}$$

For each direction there are N^2 variables $\Rightarrow$

Extra $N^2 - N$ degrees of freedom represent

dynamics of strings stretched between N 0-branes.

c.m. is represented by trace of X matrices.

QM of off-diagonal elements of matrices causes the

interaction among 0-branes.

Example: quantum fluctuations of off-diagonal elements for classically static 0-branes $\Rightarrow$ linear potential between 0-branes, just like QCD-string picture [Fatollahi, EPL 53]

Canonical momenta:

$$P_i = \frac{\partial L}{\partial X_i} = m_0\, D_t X_i \tag{3}$$

Hamiltonian:

$$H = \mathrm{Tr}\left(\frac{P_i^2}{2\, m_0} - \frac{m_0}{4\, l_s^2}\,[X_i, X_j]^2\right). \tag{4}$$

As the time-derivative of the dynamical variable A_0 is absent, its equation of motion introduces constraints, the so-called Gauss's law

$$G_a := \sum_i [X_i, P_i]_a = \mathrm{i} \sum_{i,b,c} f_{abc}\, x_{i\,b}\, p_{i\,c} \equiv 0.$$

Pair of 0-branes ($N = 2$) in 2 dim ($d = 2$)

Possible decomposition: $SU(2) \times \Lambda \times SO(2)$ [Kares, NPB **689**]

$X_{i\,a} = (\Psi)_{a\,b}(\Lambda)_{b\,j}(\eta)_{j\,i}$

Matrix Ψ: SU(2) group element $\Rightarrow$ Gauge transformations of variable $X_{i\,a}$ are captured by Ψ through ordinary gauge group left multiplications.

Parameterizing SU(2) by three Euler angles:

$\Psi = R_z(\alpha) R_x(\gamma) R_z(\beta)$,

R_a: rotation matrix about the ath axis.

Matrix η: SO(2) group element parameterized by angle $\phi \Rightarrow$ capturing effect of rotation in 2-dim space

Remaining degrees: matrix Λ

$$\Lambda = \begin{pmatrix} r\cos\theta & 0 \\ 0 & r\sin\theta \\ 0 & 0 \end{pmatrix}$$

SU(2) pure gauge degrees: α, β, $\gamma \Rightarrow$ gauging away by the Gauss law constraint:

$$G_1 = \sin\alpha \cot\gamma \; p_\alpha - \sin\alpha \csc\gamma \; p_\beta - \cos\alpha \; p_\gamma = 0$$

$$G_2 = \cos\alpha \cot\gamma \; p_\alpha - \cos\alpha \csc\gamma \; p_\beta + \sin\alpha \; p_\gamma = 0$$

$$G_3 = -p_\alpha = 0$$

p_α, p_β, p_γ: canonical momenta

$$p_\alpha = p_\beta = p_\gamma \equiv 0.$$

After imposing constraints, Hamiltonian is [Kares, NPB **689**]:

$$H = \frac{1}{2\mu}\left(p_r^2 + \frac{p_\theta^2}{r^2} + \frac{p_\phi^2}{r^2\cos^2(2\theta)}\right) + \frac{\mu}{8} r^4 \sin^2(2\theta)$$

$\mu = m_0/2$: reduced mass of relative motion

p_ϕ: constant motion

Quantum theory

$$p_\alpha \to -\mathrm{i}\,\frac{\partial}{\partial\alpha}, \qquad p_\beta \to -\mathrm{i}\,\frac{\partial}{\partial\beta}, \qquad p_\gamma \to -\mathrm{i}\,\frac{\partial}{\partial\gamma}$$

Wave-function: independent of pure-gauge degrees

(as expected!)

Laplacian

$$\nabla^2 \equiv \frac{1}{\sqrt{g}}\partial_i(\sqrt{g}\,g^{ij}\,\partial_j)$$

[Kares]

$$H = -\frac{1}{2\mu}\left(\frac{1}{r^5}\partial_r\left(r^5\partial_r\right) + \frac{1}{r^2}\nabla^2_\Omega\right) + \frac{\mu}{8}r^4\sin^2(2\theta),$$

$$\nabla^2_\Omega = \frac{1}{\sin(4\theta)}\partial_\theta\left(\sin(4\theta)\partial_\theta\right) + \frac{\partial^2_\phi}{\cos^2(2\theta)}.$$

Using scaling $\psi \to r^{-3/2}\psi$

$$H = -\frac{1}{2\mu}\left(\frac{1}{r^2}\partial_r\left(r^2\partial_r\right) + \frac{1}{r^2}(\nabla^2_\Omega - {}^{15}\!/\!_4)\right) + \frac{\mu}{8}r^4\sin^2(2\theta),$$

$$\nabla^2_\Omega \, \mathcal{Y}_\lambda(\theta,\phi) = \lambda \, \mathcal{Y}_\lambda(\theta,\phi)$$

$$\mathcal{Y}_\lambda(\theta,\phi) = g_\lambda(\theta) \frac{e^{i m_z \phi}}{\sqrt{2\pi}}$$

m_z: $0, \pm 2, \pm 4, \cdots$ (due to Z_2 sym)

New variable: $x = \cos(4\theta), 0 \leq \theta \leq \pi/4$

$$\frac{\mathrm{d}}{\mathrm{d}x}\left((1-x^2)\frac{\mathrm{d}g_\lambda}{\mathrm{d}x}\right) - \frac{m^2}{2(1+x)} g_\lambda(x) = \frac{\lambda}{16}\, g_\lambda(x).$$

$$(1-x^2)Q''(x)+\big(m-(m+2)x\big)Q'(x)$$

$$-\left(\lambda+\frac{m(m+2)}{4}\right)Q(x)=0,$$

Solution: Jacobi polynomials of order

$n=l-m\geq 0$, $\mathcal{P}_n^{(0,m)}(x)$

$$\lambda=-16(l-m/2)(l-m/2+1), \qquad m\leq l=0,1,\cdots,$$

Normalized Ang. Mom. eigenfunctions

$$\mathcal{Y}_l^m(\theta,\phi) = \sqrt{\frac{2l-m+1}{2^{m+1}}}(1+\cos(4\theta))^{m/2}\mathcal{P}_{l-m}^{(0,m)}(\cos(4\theta))$$

Recurrence relations:

$$\frac{2(l+1)(l-m+1)}{(2l-m+1)(2l-m+2)}\mathcal{P}_{l-m+1}^{(0,m)}(x)+\frac{2l(l-m)}{(2l-m)(2l-m+}$$

$$+\frac{m^2}{(2l-m)(2l-m+2)}\mathcal{P}_{l-m}^{(0,m)}(x) = x\,\mathcal{P}_{l-m}^{(0,m)}(x)$$

Need a basis-function: H.O. (as usual!)

$$\psi_{E,l,m}(r,\theta,\phi) = R_{E,l,m}(r)\, \mathcal{Y}_l^m(\theta,\phi)$$

Radial eq.

$$-\frac{1}{2\mu}\left(R''_{E,l,m} - \frac{J_l^m(J_l^m+1)}{r^2} R_{E,l,m}\right) + \frac{1}{2}\mu r^2 R_{E,l,m}$$

$$= E\, R_{E,l,m}$$

$$J_l^m = 4\,l - 2\,m + 3/2.$$

$$R_{k,l,m}(r) = \sqrt{\frac{2\,k!\,\mu^{J_l^m+3/2}}{\Gamma(k+J_l^m+3/2)}}\; r^{J_l^m} e^{-\mu r^2/2}\, L_k^{(J_l^m+1/2)}(\mu r^2)$$

$$k = 0, 1, 2, \cdots$$

$$E_{k,l,m} = 2k + J_l^m + 3/2 = 2k + 4l - 2m + 3$$

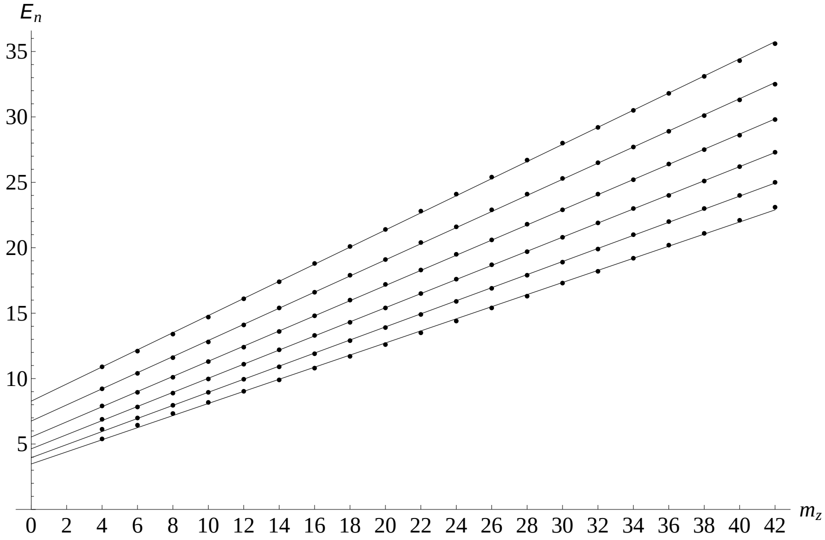
Basis function: 80 per Ang. Mom.

Rescalings:

$$X_i \to g_s^{1/3}\, l_s\, X_i, \qquad P_i \to g_s^{-1/3}\, l_s^{-1}\, P_i$$

$$E \propto \, g_s^{1/3}\, l_s^{-1}$$

m_z	E_1	E_2	E_3	E_4	E_5	E_6	m_z	E_1	E_2	E_3	E_4	E_5
0	2.66	4.54	5.95	7.15	8.25	9.09	22	13.5	14.9	16.5	18.3	20.
2	4.13	5.31	6.22	7.16	8.34	9.79	24	14.4	15.9	17.6	19.5	21.
4	5.39	6.13	6.89	7.91	9.22	10.9	26	15.4	16.9	18.7	20.6	22.
6	6.44	6.99	7.83	8.95	10.4	12.1	28	16.3	17.9	19.7	21.8	24.
8	7.33	7.96	8.89	10.1	11.6	13.4	30	17.3	18.9	20.8	22.9	25.
10	8.18	8.95	9.97	11.3	12.8	14.7	32	18.2	19.9	21.9	24.1	26.
12	9.03	9.95	11.1	12.4	14.1	16.1	34	19.2	21.0	23.0	25.2	27.
14	9.90	10.9	12.2	13.6	15.4	17.4	36	20.2	22.0	24.0	26.4	28.
16	10.8	11.9	13.3	14.8	16.6	18.8	38	21.1	23.0	25.1	27.5	30.
18	11.7	12.9	14.3	16.0	17.9	20.1	40	22.1	24.0	26.2	28.6	31.
20	12.6	13.9	15.4	17.2	19.1	21.4	42	23.1	25.0	27.3	29.8	32.


E_n
35
30
25
20
15
10
5
0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34 36 38 40 42
m_z

$$E_1 = 3.474\,[0.059] + 0.462\,[0.002]\, m_z$$

$$E_2 = 3.953\,[0.031] + 0.500\,[0.001]\, m_z$$

$$E_3 = 4.632\,[0.020] + 0.539\,[0.001]\, m_z$$

$$E_4 = 5.535\,[0.027] + 0.579\,[0.001]\, m_z$$

$$E_5 = 6.754\,[0.038] + 0.616\,[0.001]\, m_z$$

$$E_6 = 8.277\,[0.047] + 0.654\,[0.002]\, m_z$$

$$m_z : 0, 2, 4, \cdots, 42$$

Less than %2 error: staight-lines fit data!

Thank you on behalf of Matrix Coordinates!