Black holes, hidden symmetries and complete integrability

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XVIII Int. Conference “Geometry, Integrability and Quantization, Varna, June 3-8, 2016

“Everything should be made as simple as possible, but not simpler.”

— Albert Einstein

Higher Dimensional BHs

Motivations :

(1) Extra - dimensions and string theory

(2) Brane - world models

(3) Black holes as probes of extra dimensions

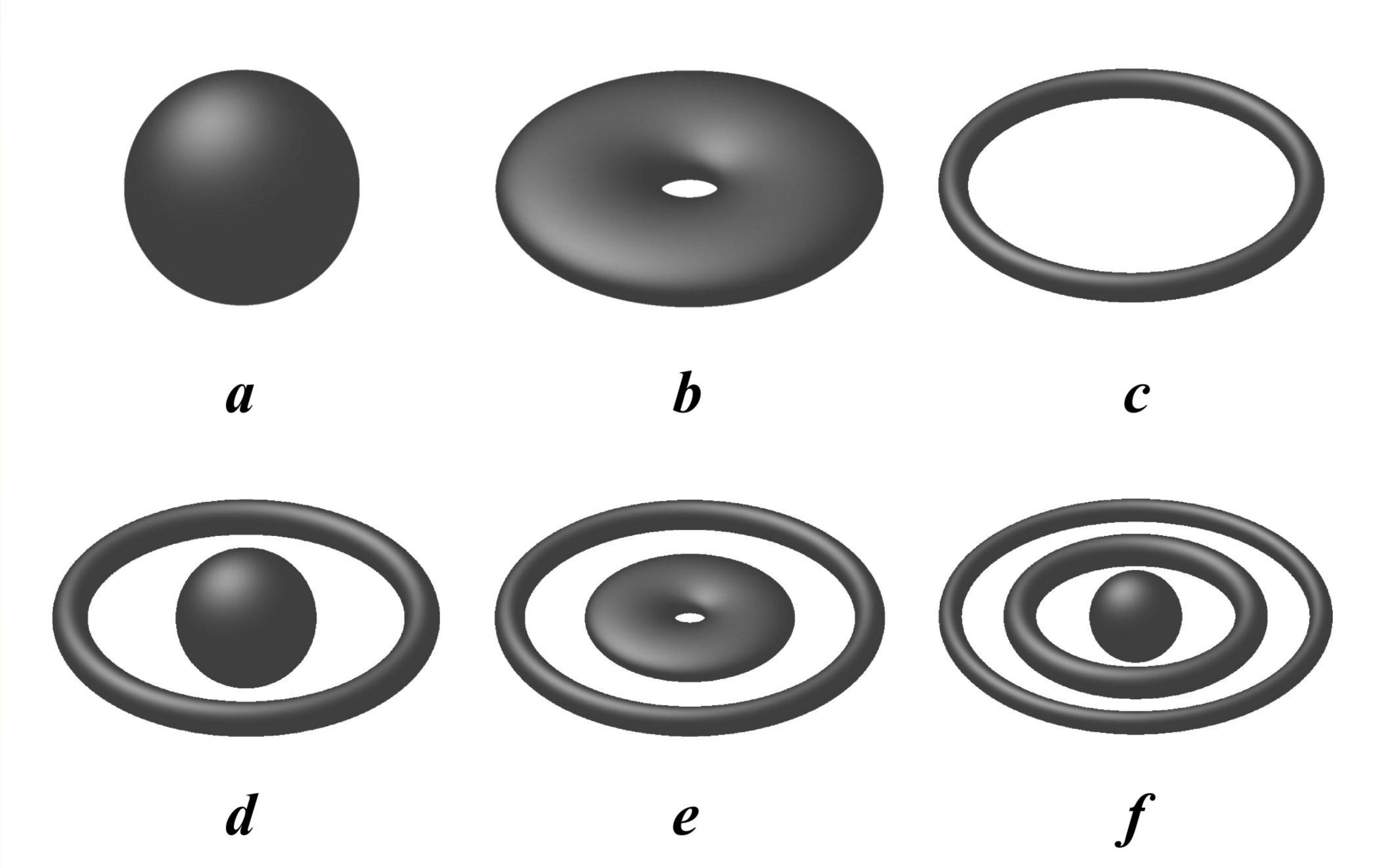
(4) Micro BHs production in colliders?

(5) Generic and non - generic properties of BHs

Two big "surprises" in study of HD black holes:

1. The topology of the horizon can be more complicated than the topology of the sphere S^{D-2}. 5D exact solutions for stationary black rings (Emparan and Reall, 2002) plus many later publications. Stability of such solutions?
2. Properties of HD stationary black holes with spherical topology of the horizon are quite similar to the properties of their 4D "cousins" (Kerr metric): geodesic equations are completely inegrable and wave equations are completely separable (V.F. and Kubiznak, 2007, plus Alberta separatists' later publications)

5D vac. stationary black holes



In this talk I shall focus on the "second big surprize":

Hidden Symmetries and Complete Integrability in HD BHs.

1. Brief history;
2. Liouville theory of the complete integrability;
3. Origin and properties of the hidden symmetries;
4. Killing tensors and Killing towers;
5. Applications and results;
6. Recent developments.

Remark: Whenever I tell a black hole, it means that I discuss an isolated higher dimensional stationary rotating black hole with the spherical topology of horizon in a ST, which asympotically is either flat or (Anti)DeSitter. Its metric is a solution of the Einstein equations in $D = 2n + \varepsilon$ dimensions

$$R_{ab} = \Lambda g_{ab}$$

Main Message: Properties of higher dimensional rotating BHS and the 4D Kerr metric are very similar.

HDBHs give a new wide (infinite) class of completely integrable dynamicalsystems.

Higher Dimensional Black Holes

Tangherlini '63 metric (HD Schw. analogue)

....

Myers & Perry '86 metric (HD Kerr analogue)

....

Kerr - NUT - AdS '06 (Chen, Lu, and Pope;

The most general HD BH solution)

"General Kerr-NUT-AdS metrics in all dimensions“, Chen, Lü and Pope, Class. Quant. Grav. 23 , 5323 (2006).

$$n=\left[D/2\right],\ \ D=2n+\varepsilon$$

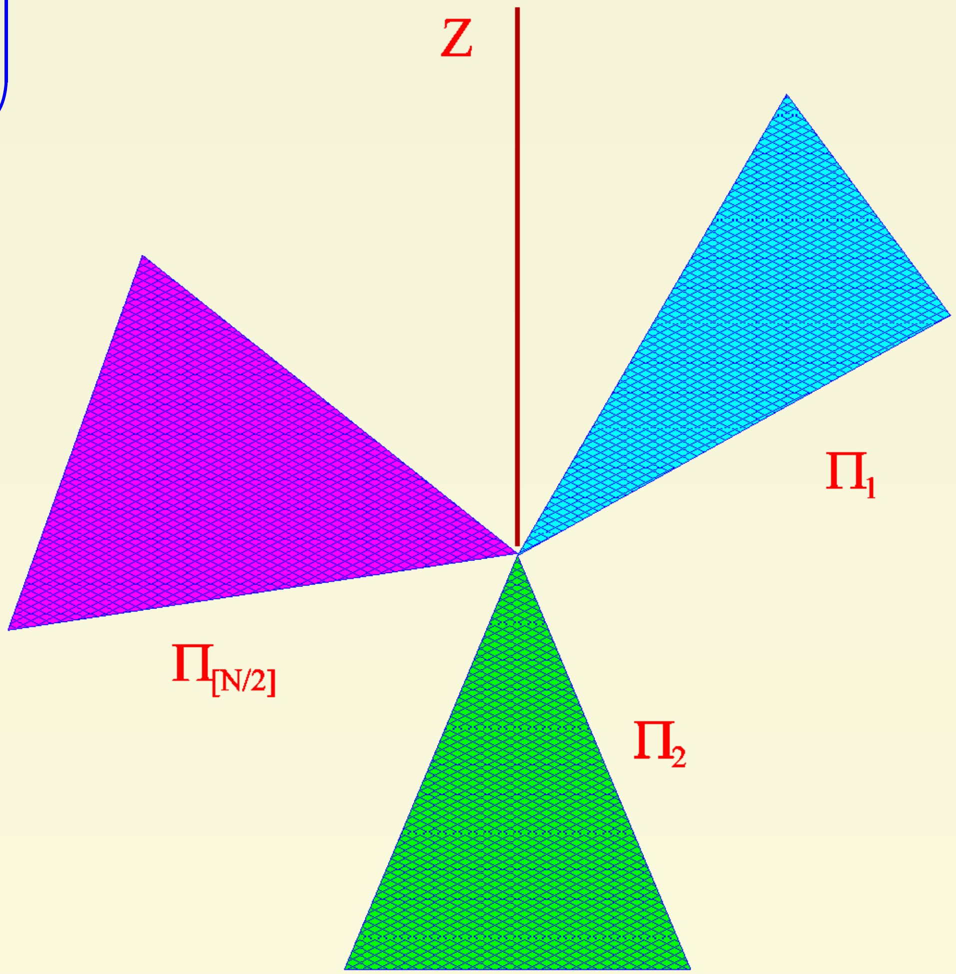
$$R_{ab}=(D-1)\lambda g_{ab}$$

λ, M – mass, a_k – $(n-1+\varepsilon)$ rotation parameters,

M_α – $(n-1-\varepsilon)$ `NUT' parameters

$\boxed{\textit{Total \# of parameters is } D-\varepsilon}$

$$\begin{pmatrix} \begin{pmatrix} 0 & J_1 \\ -J_1 & 0 \end{pmatrix} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \begin{pmatrix} 0 & J_n \\ -J_n & 0 \end{pmatrix} \end{pmatrix}$$



Stationary - Killing vector ξ_t;

Axisymmetric - $(n - 1 + \varepsilon)$ Killing vectors ξ_{ψ_μ};

When cosmological constant λ and NUT parameters vanish one has Myers - Perry metric (1986)

Generator of Symmetries

Principal Closed Conformal KY Tensor

2 - form $h_{[\mu\nu]}$ with the following properties :

(i) Non - degenerate (maximal matrix rank, $2n$)

(ii) Closed $dh = 0$

(iii) Conformal KY tensor : $\nabla_c h_{ab} = g_{ca}\xi_b - g_{cb}\xi_a$,

$\xi_a = \frac{1}{D\text{-}1}\nabla^b h_{ba}$, ξ_a is a primary Killing vector

For briefness, we call this object a "Principal Tensor"

All Kerr-NUT-AdS metrics
in any number of ST dimensions
possess a PRINCIPAL TENSOR
(V.F.&Kubiznak '07)

Uniqueness Theorem

A solution of Einstein equations with the cosmological constant, which possesses a PRINCIPAL TENSOR is a Kerr-NUT-AdS metric

(Houri,Oota&Yasui ’07 ’09; Krtous, V.F. .&Kubiznak ’08;)

Geodesic equations in ST with a PRINCIPAL TENSOR are completely integrable in Liouville sense. Important field equations allow a complete separation of variables. ("Alberta separatists")

In the rest of the talk I shall explain why it happens.

BRIEF REMARKS ON COMPLETE INTEGRABILITY

Hamiltonian system

Phase space : Differential manifold M^{2D} with

a symplectic form Ω (non-denerate rank 2 closed, $d\Omega = 0$).

Observables are scalar functions on M^{2D}.

$X_F = \Omega^{-1} dF$ is a Hamiltonian vector field associated

with an observable F.

The dynamical equations for the Hamiltonian H are $\frac{dz}{dt} = X_H$.

Poisson bracket $\{F,Q\} = dF\,\Omega^{-1} dQ = [X_F, X_Q]$.

Integral of motion $\dot{F} \equiv \{F,H\} = 0$.

Integrals of motion F,Q are in involution if $\{F,Q\} = 0$

Darboux's theorem: Suppose that Ω is a simplectic 2-form on an $n=2m$ dimensional manifold M. Then in a neighrhood of each point q on M, there is a coordinate chart U in which $\Omega = d\theta = dq_1 \wedge dp_1 + \ldots + dq_m \wedge dp_m$.

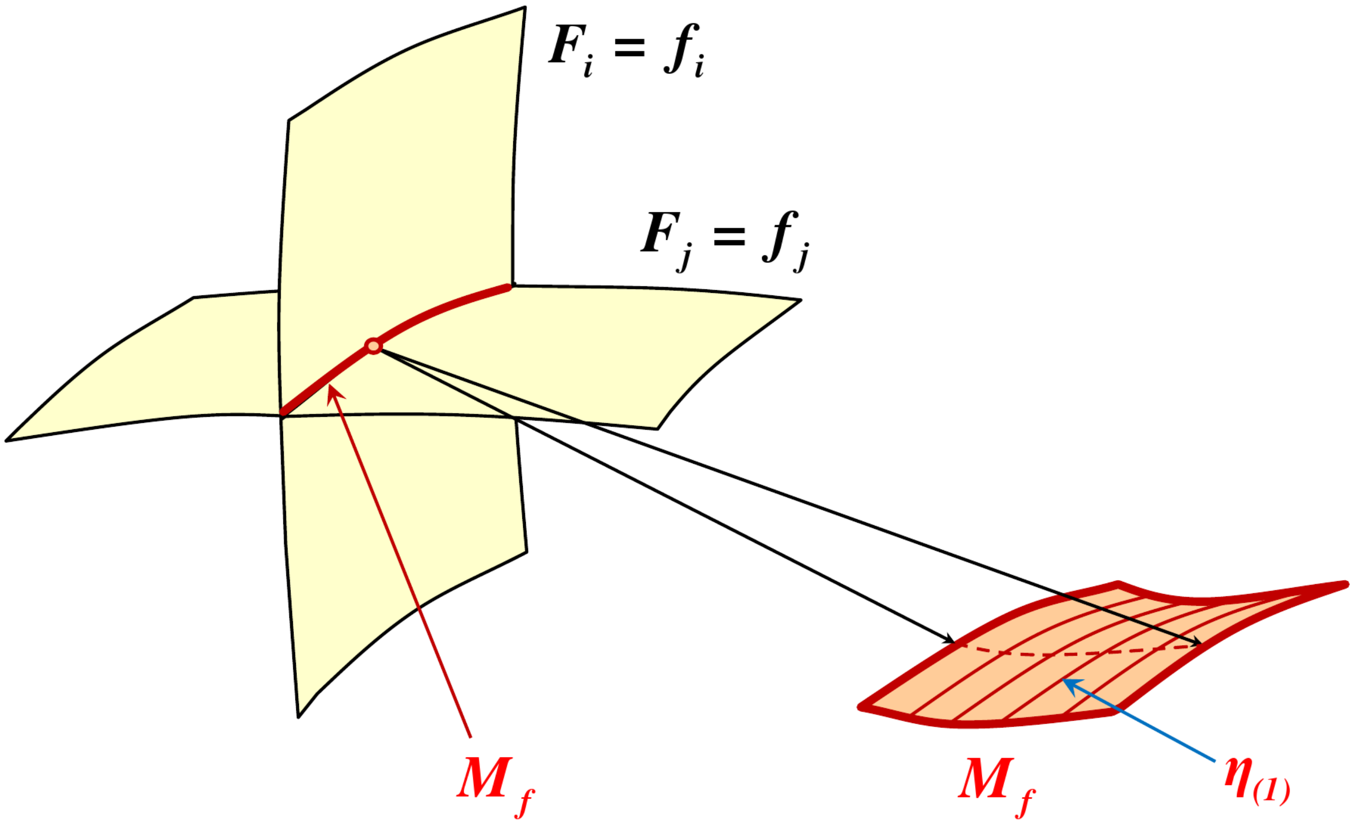
U is called a Darboux chart. The manifold can be covered by such charts (Darboux atlas).

Liouville theorem: Dynamical equations in 2D dimensional phase space are completely integrable if there exist D independent commuting integrals of motion.

General idea: D commuting independent integrals of motion F_i can be used as coordinates on the phase space.

Denote by M_f the level set for $\{F_i\}$. Gradients dF_i are linearly independent $\Rightarrow M_f$ is D dimensional submanifold of M^{2D}.

The involution condition implies that $\Omega_{\mathrm{ij}}\,|_{L_F} = X_{F_i}\,\Omega\,X_{F_j} = 0$, hence M_f is a Lagrangian submanifold. The vector fields X_{F_i} are tangent to it and mutually commute.



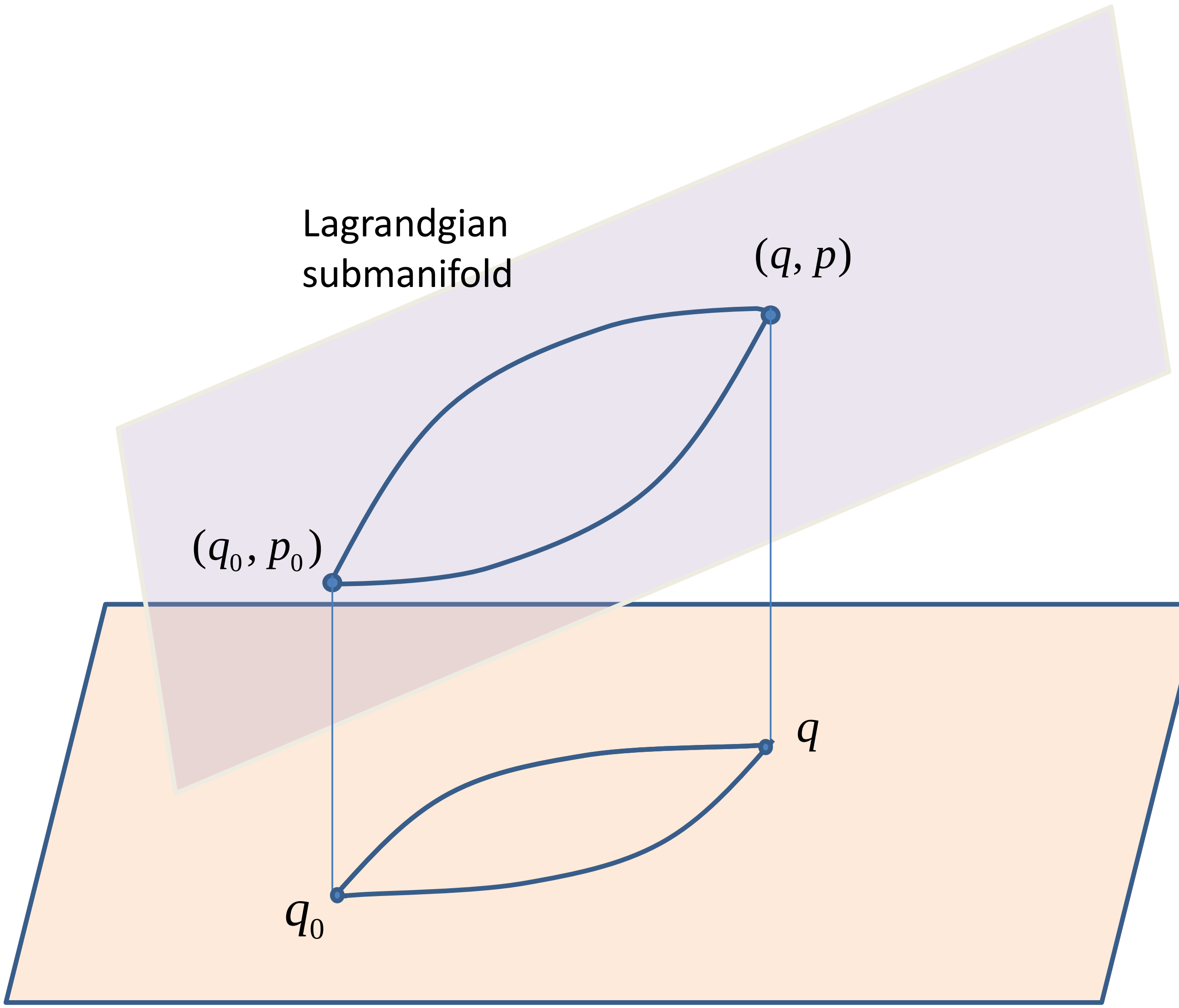
$F_i = f_i$
$F_j = f_j$
M_f
M_f
$\eta_{(1)}$

Denote $\Omega = d\theta$ and consider the function $W(q,P) = \int_{m_0}^{m} \theta \,|_{M_f}$.

m is a point of the phase space, and q are its coordinates.

Since $d\Omega = 0$ on M_f, the integral does not depend on a choice of the path. $W(q,P)$ can be used as a generating function, which produces a canonical transformation from the original (q,p) to new canonical coordinates $(Q^i, P_i \equiv F_i)$: $Q^i = \dfrac{\partial W(q,P)}{\partial P_i}$. In these new coordinates the Hamilton's equations become trivial:

$$\dot{P}_i = \{P_i, H\} = 0, \quad \dot{Q}^i = \{Q^i, H\} = \frac{\partial H}{\partial P_i} = const.$$


Lagrandgian submanifold
(q, p)
(q_0, p_0)
q
q_0

Relativistic Particle as a Dynamical System

Preferable coordinates in the phase space:

$$(x^a, p_a = g_{ab}\dot{x}^b), \quad H(p,x) = \tfrac{1}{2} g^{ab} p_a p_b .$$

Monomial in momenta integrals of motion

$\mathcal{K} = K^{a\ldots b} p_a \ldots p_b$ imply that $K_{a\ldots b}$ is a Killing tensor:

$$K_{(a\ldots b;c)} = 0$$

$$\{\mathcal{K}_1, \mathcal{K}_2\} = 0 \Leftrightarrow \quad [K_1, K_2] = 0.$$

Motion of particle in D-dimensional ST is completely integrable if there exist D independent commuting Killing tensors (and vectors)

Metric is a best known example of rank 2 Killing tensor

If $\mathcal{K}_{(n)}$ and $\mathcal{K}_{(m)}$ are 2 monomial integrals of order n and m, then $\mathcal{K}_{(n)} \bullet \mathcal{K}_{(m)}$ is a monomial integral of order $n+m$. The corresponding Killing tensor is called reducible.

Properties of 4D black holes

$KT \quad (K_{(ab;c)} = 0)$

Walker & Penrose '70

$KY = \sqrt{KT} \; (f_{a(b;c)} = 0)$

Penrose & Floyd '73

Carter '68
H-J separation
4th integral of
motion

$$PCCKY \; (h = {*f}, h = db)$$

$$\nabla_{(a} h_{b)c} = g_{ab}\xi_c - g_{c(a}\xi_{b)}$$

$$\xi_a = \tfrac{1}{3} h_{ba}{}^{;b}$$

$\xi = \xi_{(t)}$ a is primary KV

$\zeta^a = K^{ab}\xi_b$ is a SKV

Hughston & Sommers '75

Einstein space $(R_{ab} = \Lambda g_{ab})$

which admits PCCKY is either

Kerr - NUT - AdS or C - metric

Parallel transport : Consider a geodesic, and let p^a be a tangent vector. Let k_{ab} be a Killing-Yano tensor: $k_{a(b;c)}=0$. Then a vector $q_a=k_{ab}p^b$ is parallel propagated. Really,

$$p^c q_{a;c}=k_{ab;c}p^b p^c+k_{ab}p^b{}_{;c}p^c=0.$$

A 2-plane, determined by a 2-form σ=*(p $\wedge$ q) is also parallel propagated along a geodesic. In order to find 2 parallel propagated vectors, that span it it is sufficient to solve a simple ODE (Marck, 1983).

Tidal disruption of a white dwarf by a massive black hole

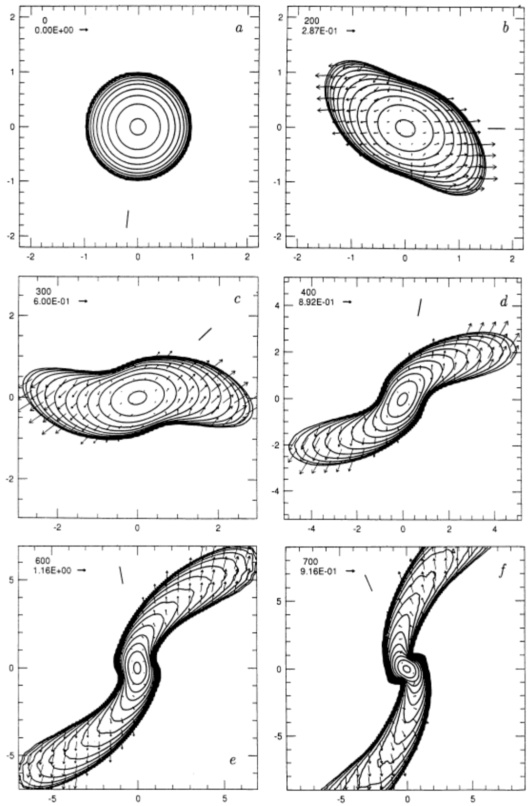
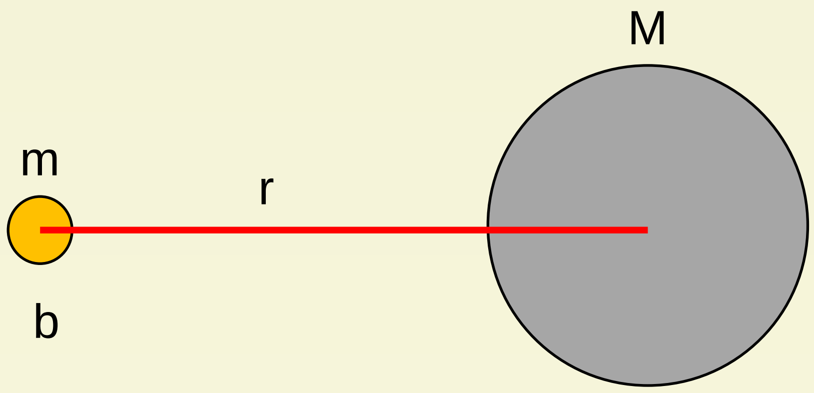


FIG. 1.—Density contours in the equatorial (X-Y) plane, and the velocity field for the encounter N1. Results are presented in units which $G = M_{wd} = R_{wd} = 1$. The time step number and the velocity scale are shown in the upper left corner of each frame. The innermost contour corresponds to 90% of the central density, and at successive contours the density decreases by a factor of 2. The short thick line in each frame indicates the direction to the black hole (BH). Frames are obtained in two runs with different spatial resolution. Frames (*a*)–(*e*) are computed on the uniformly expanding grid. Frames (*f*)–(*i*) are computed on the grid the expansion of which was stopped in order to get better resolution of the stellar core. The frame (*j*) belongs to the first run, and shows the geometry of the jets at the end of the computations. The core is not well resolved in this frame. The two last frames, (*l*) and (*k*), show the density contours and the velocity field in the Y-Z and X-Z planes, passing through the center of the star, for the moment of time corresponding to frame (*f*). Times corresponding to the various frames are (*a*) −4.87, (*b*) 0, (*c*) 1.35, (*d*) 3.24, (*e*) 5.34, (*f*) 8.18, (*g*) 12.52, (*h*) 15.34, (*i*) 17.62, and (*j*) 38.95.

«Relativistic tidal interaction of a white dwarf with a massive black hole»
Frolov, V. P., Khokhlov, A. M., Novikov, I. D., & Pethick, C. J.
Astrophysical Journal, vol. 432, p. 680-689, (1994)



Condition of a static disruption:

$$\frac{\mathrm{GMb}}{\mathrm{r}^3} = \frac{Gm}{b^2} \rightarrow r = (M/m)^{1/3} b$$

Higher dimensions

Denote $D = 2n + \varepsilon$.

D - dimensional AdS - Kerr - NUT metric has $n + \varepsilon$ Killing vectors. For complete integrability of geodesic equations one needs n more integrals of motion.

GENERAL SCHEME

Forms (=AStensor)

(1) External product: $\alpha_q \wedge \beta_p = (\alpha \wedge \beta)_{q+p}$

(2) Hodge dual: $*(\alpha_q) = (*\alpha)_{D-q}$

(3) External derivative: $d(\alpha_q) = (d\alpha)_{q+1}$

(4) Closed form: $d(\alpha_q) = 0$ (locally $\alpha_q = d(\beta_{q-1})$)

Killing-Yano family

Let ω be p - form on the Riemannian manifold.

Then its Hodge dual $*\omega$ is $(D - p)$- form.

Two operations: derivative d and coderivative δ.

$\nabla\omega = d\omega + g\bullet\delta\omega + (\ldots) \qquad (\delta = *d*)$

If $(\ldots) = 0$ ω is a conformal KY tensor;

If $\delta\omega = (\ldots) = 0$ ω is a KY tensor;

If $d\omega = (\ldots) = 0$ ω is a closed conformal KY tensor.

Let $f_{\mu_1\mu_2\ldots\mu_n}$ be a Killing - Yano tensor $f_{\mu_1\mu_2\ldots(\mu_n;\nu)} = 0$, then

$f_{\mu_1\mu_2\ldots\mu_n} p^{\mu_n}$ is a parallel propagated along a geodesic form;

$K_{\mu\nu} = f_{\mu\mu_2\ldots\mu_n} f_{\nu}{}^{\mu_2\ldots\mu_n}$ is the Killing tensor

$K^{\mu\nu} p_\mu p_\nu$ is an integral of geodesic motion

Integrability conditions of the KY equation:

$$\nabla_a \nabla_b f_{c_1 c_2 \ldots c_p} = \frac{p+1}{2} R_{ea[bc_1} f^{e}{}_{c_2 \ldots c_p]}$$

Maximal number of independent KY tensor

of rank p is $\dfrac{(D+1)!}{(D-p)!(p+1)!}$

Maximal number of independent CCKY tensor

of rank p is $\dfrac{(D+1)!}{(D-p+1)!p!}$

Example : Killing vector $\xi_{(a;b)} = 0, \quad \xi_{a;b;c} = R_{abc}{}^{f}\xi_{f}$

Maximum number of independent Killing vectors :

$$N_D[\xi] = D + \frac{1}{2}D(D-1) = \frac{1}{2}D(D+1)$$

Flat sacetime (as well as (anti)deSitter space) has the maximum number of Killing-Yano and closed conformal Killing-Yano tensors. The Killing-Yano *n*-forms in the flat spacetime allow quite simple description. Consider a set of *n* integer numbers

$$\{a_1,\dots,a_n\},\quad 1\le a_1\le\dots\le a_n\le D$$

Translationally invariant Killing-Yano *n*-forms

$$k^{\{a_1,\dots,a_n\}}=dx^{a_1}\wedge dx^{a_2}\wedge\dots\wedge dx^{a_n}$$

Consider another set of $n+1$ integer numbers

$$\{a_1,\dots,a_{n+1}\},\quad 1\le a_1\le\dots\le a_n\le a_{n+1}\le D$$

Rotational Killing-Yano *n*-forms

$$\hat{k}^{\{a_1,\dots,a_{n+1}\}}=x^{[a_1}\wedge dx^{a_2}\wedge\dots\wedge dx^{a_n+1]}$$

Closed conformal Killing-Yano tensors are dual of the above objects.

Properties of CKY tensor

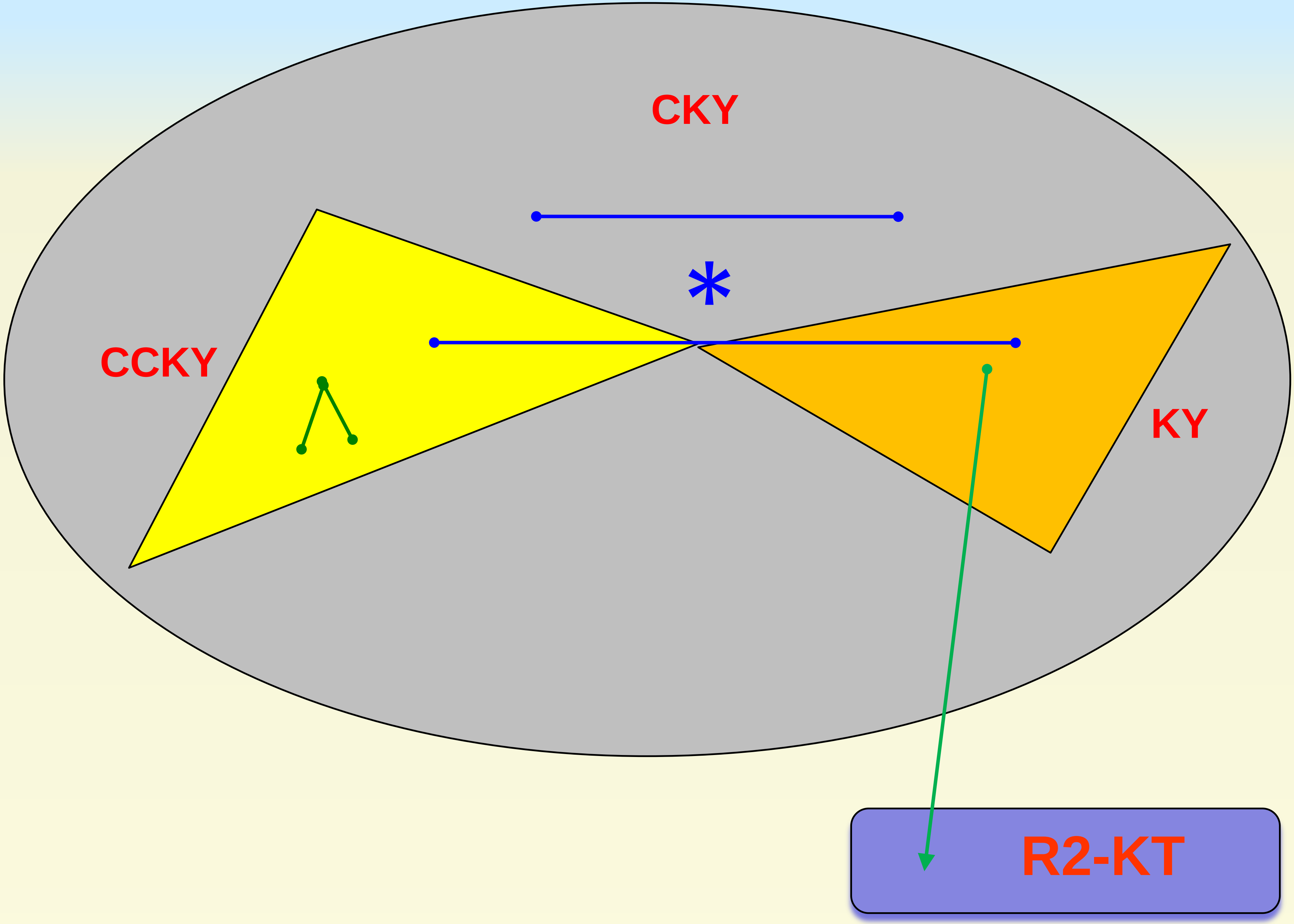
Hodge dual of CKY tensor is CKY tensor

Hodge dual of CCKY tensor is KY tensor;

Hodge dual of KY tensor is CCKY tensor;

External product of two CCKY tensors is a CCKY tensor

(Krtous,Kubiznak,Page &V.F. '07; V.F. '07)


CKY
CCKY
KY
R2-KT

Principal Tensor

External powers of CCKY tensor give new CCKY. That is why they are "better" as symmetry generators than KY tensors.

For a 1-form ξ one has $\xi \wedge \xi$=0. For higher than rank 1 forms their external powers are, generally, non-vanishing.

The lower rank of the CCKY form, the more non-trivial new CCKY tensors it can generate. The rank 2 CCKY form is in this sence the most promicing.

In order to generate the largest number of non-vanishing CCKY forms, a 2-form of the CCKY object must have the highest possible matrix rank (2*n*).

The Principal CCKY tensor (or simply the Principal Tensor) is a non-denerate 2-form which is CCKY object. (Some additional requirements will be added later.)

Killing-Yano Tower

Killing-Yano Tower

$$CCKY: \quad h \;\Rightarrow\; h^{\wedge j} = \underbrace{h \wedge h \wedge \ldots \wedge h}_{j\ times}$$

KY tensors: $k_j = {*h^{\wedge j}}$

Killing tensors: $K^j = k_j \bullet k_j$

Primary Killing vector: $\xi_a = \frac{1}{D-1} \nabla^b h_{ba}$

Secondary Killing vectors: $\xi_j = K_j \bullet \xi$

Total number of conserved quantities:

$$(n+\varepsilon)+(n-1)+1=2n+\varepsilon=D$$

KV KT g

The integrals of motion are functionally independent and in involution. The geodesic equations in the AdS-Kerr-NUT ST are completely integrable.

Canonical Coordinates

$$h\left(m_{\pm}^{\mu}\right) = \mp i x^{\mu} m_{\pm}^{\mu} \qquad m_{\pm}^{\mu} = e^{\mu} \pm i e^{\hat{\mu}}$$

We include in the definition of the Principal Tensor

h the following requirement:

The 2-form *h* in the ST with $D = 2n + \varepsilon$ dimensions

has *n* non-equal independent eigen-values x^{μ}, so

that there exists n (mutually orthogonal) two-planes.

Canonical coordinates: n essential coordinates x^{μ} and $n+\varepsilon$ Killing coordinates ψ_j. Total number of such "canonical" coordinates is $D=2n+\varepsilon$.

$$g=\sum_{\mu}(e^{\mu}e^{\mu}+e^{\hat{\mu}}e^{\hat{\mu}})+\varepsilon e^{n+1}e^{n+1},\ h=\sum_{\mu}x^{\mu}e^{\mu}\wedge e^{\hat{\mu}}$$

The components of Killing tower objects in such a basis are polynomials in x^{μ}.

“Off-shell” metrics, which admit the Principal Tensor, allow complete description

$$g_{ab} = \sum_{\mu} (e_a^{\mu} e_b^{\mu} + e_a^{\hat{\mu}} e_b^{\hat{\mu}}) + \varepsilon e_a^{n+1} e_b^{n+1},$$

$$e^{\mu} = \frac{1}{\sqrt{Q_{\mu}}} dx_{\mu}, \quad e^{\hat{\mu}} = \sqrt{Q_{\mu}} \sum_{i=0}^{n-1} A_{\mu}^{(i)} d\psi_i$$

$$Q_{\mu} = \frac{X_{\mu}}{U_{\mu}}, \quad , \quad U_{\mu} = \prod_{\nu \neq \mu} (x_{\nu}^2 - x_{\mu}^2), \quad X_{\mu} = X_{\mu}(x_{\mu})$$

$$\prod_{\nu=1}^{n}(1+tx_\nu^2)=\sum_{j=0}^{n}t^j A^{(j)}\,,$$

$$(1+tx_\mu^2)^{-1}\prod_{\nu=1}^{n}(1+tx_\nu^2)=\sum_{k=0}^{n-1}t^k A_\mu^{(k)}\,.$$

Arbitrary functions $X_\mu(x^\mu)$ after substitution into Einstein equations become polynomials. Their coefficients are just papameters of the metric.

Houri, Oota, and Yasui [PLB (2007); JP A41 (2008)] proved this result under additional assumptions: $L_\xi g=0$ and $L_\xi h=0$. Later Krtous, V.F., Kubiznak [arXiv:0804.4705 (2008)] and Houri, Oota, and Yasui [arXiv:0805.3877 (2008)] proved this without additional assumptions.

Solutions of the Einstein equations with the cosmological constant ("On-shell" metrics)

$$X_\mu = b_\mu x_\mu + \sum_{k=0}^{n} c_\mu x_\mu^{2k} \quad \text{for } D \text{ even.}$$

A similar expression for D odd.

This is nothing but the Kerr-NUT-(A)dS metric, written in special (canonical) coordinates.

Intermediate Summary

1. The Principal Tensor (if it exists) generates the Killing tower, which roughly contains a half of the Killing vectors and a half of the Killing tensors. The Killing vectors are responcible for explicite symmetry of the spacetime, while the Killing tensors describe its hidden symmetries.
2. Eigen-values of the Principal Tensor together with the Killing parameters, determined by the Killing vectors provide one with special coordinates, in which the metric and the PT has "simple" canonical form.
3. Equations for the Principal Tensor and their integrability conditions, written in the canonical coordinates, allows one to find the (off-shell) metric.
4. After imposing the Einstein equations, this metric becomes Kerr-NUT-(A)dS solution.

Separation of variables

This is a very special property, which depends both on the type of the equaion and special choice of the coordinates.

Hamilton-Jacobi $(\nabla S)^2 = m^2$

$\Updownarrow$ WKB $\Phi \sim \exp(iS)$

Klein-Gordon $(\Box - m^2)\Phi = 0$

$\Updownarrow$ "Dirac eqn=$\sqrt{\text{KG eqn}}$ "

Dirac equation $(\gamma^{\mu}\nabla_{\mu} + m)\Psi = 0$

Separation of variables in HJ eqs

For the Hamiltonian

$H(P,Q),\ P = p_1,\ldots,p_m,\ Q = q_1,\ldots,q_m,$

the Hamilton-Jacobi equation is

$H(\partial_P S, Q) = 0.$

Suppose q_1 and $\partial_{q_1} S$ enter this equation as $\Phi_1(\partial_{q_1} S, q_1)$.

Then the variable q_1 can be separated:

$S = S_1(q_1, C_1) + S'(q_2,\ldots,q_m),$

$\Phi_1(\partial_{q_1} S, q_1) = C_1,$

$H_1(\partial_{q_2} S',\ldots,q_2,\ldots;C_1) = 0$

Complete separation of variables:

$$S = S_1(q_1, C_1) + S_2(q_2, C_1, C_2) + \ldots S_m(q_m, C_1, \ldots, C_m).$$

The constants C_i generate first integrals on the phase space. When these integrals are independent and in involution the system is integrable in the Liouville sence.

Separability of the Hamilton–Jacobi equation in canonical coordinates

$$\frac{\partial S}{\partial \lambda} + g^{ab}\, \partial_a S\, \partial_b S = 0$$

$$S = -w\lambda + \sum_{\mu=1}^{n} S_\mu(x_\mu) + \sum_{k=0}^{m} \Psi_k \psi_k$$

$$(S_\mu')^2 = -\frac{1}{X_\mu^2}\Big(\sum_{k=0}^{m}(-x_\mu^2)^{n-1-k}\Psi_k\Big)^2 + \frac{1}{X_\mu}\sum_{k=0}^{m}\tilde{c}_k(-x_\mu^2)^{n-1-k}$$

V. F., P. Krtous , D. Kubiznak , JHEP 0702:005 (2007)

Separability of the Klein–Gordon equation

,

$$(\Box-\mu^2)\Phi=0 \qquad \Phi=\prod_{\mu=1}^{n}R_\mu(x_\mu)\prod_{k=0}^{m}e^{i\Psi_k\psi_k}.$$

$$(X_\mu R_\mu)'+\varepsilon\frac{X_\mu}{x_\mu}R_\mu-\frac{R_\mu}{X_\mu}\left(\sum_{k=0}^{m}(-x_\mu^2)^{n-1-k}\Psi_k\right)^2-\sum_{k=0}^{m}b_k(-x_\mu^2)^{n-1-k}R_\mu=0$$

V. F., P. Krtous , D. Kubiznak , JHEP 0702:005 (2007)

Weakly charged higher dimensional rotating black holes

Hamiltonian $$H = \frac{1}{2} g^{ab} (p_a - qA_a)(p_b - qA_b)$$

HJ equation $$-\mu^2 = g^{ab} \left[\left(\frac{\partial S}{\partial x^a} - qA_a\right)\left(\frac{\partial S}{\partial x^b} - qA_b\right) \right]$$

Klein-Gordon equation

$$\left[g^{ab} (\nabla_a - iqA_a)(\nabla_b - iqA_b) - \mu^2 \right] \Phi = 0$$

$$F_{ab}{}^{;b} = 0 \quad \leftrightarrow \quad A^a{}_{;a} = 0 \quad \leftrightarrow \quad A^{a;b}{}_{;b} = 0$$

$$\xi^{a;b}{}_{;b} = 0 \quad \leftrightarrow \quad A_a = Q\xi_a \ \text{(in Ricci flat)}$$

$$g^{ab}\frac{\partial S}{\partial x^a}\frac{\partial S}{\partial x^b} + M^2 = 0,$$

$$[\Box - M^2]\Phi = 0,$$

$$M^2 = \mu^2 - 2e\Psi_0 + e^2\xi^2_{(0)}$$

For a primary Killing vector field one again has a complete separation of variables

[V.F. and Krtous, 2011]

Complete separation of variables in KG and HJ eqns in Kerr-NUT-AdS ST (V.F.,Krtous&Kubiznak ’07)

Separation constants KG and HJ eqns and integrals of motion (Sergyeyev&Krtous ‘08)

Separation of variables in Dirac eqns in Kerr-NUT-AdS metric (Oota&Yasui ‘08, Cariglia, Krtous&Kubiznak ’11)

Separability of gravitational perturbations in Kerr-NUT-(A)dS Spacetime (Oota andYasui '10)

Metrics admitting a principal Killing-Yano tensor with torsion (Houri, Kubiznak, Warnick and Yasui '12)

Parallel transport along timelike geodesics

Let u^a be a vector of velocity and h_{ab} be a PCKYT.

$P_a^b = \delta_a^b + u_a u^b$ is a projector to the plane orthogonal to u^a.

Denote $F_{ab} = P_a^c P_b^d h_{cd} = h_{ab} + u_a u^c h_{cb} + h_{ac} u^c u_b$

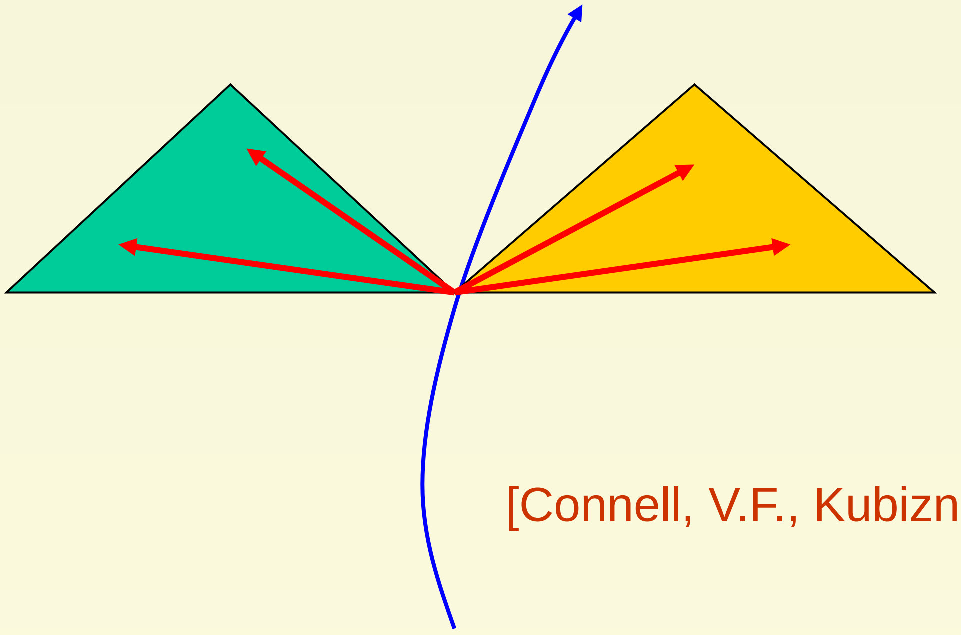
Lemma (Page): F_{ab} is parallel propagated along a geodesic:

$$\nabla_u F_{ab} = 0$$

Proof: We use the definition of the PCKYT

$$\nabla_u h_{ab} = u_a \xi_b - \xi_a u_b$$

Suppose h_{ab} is a non-degenerate, then for a generic geodesic eigen spaces of F_{ab} with non-vanishing eigen values are two dimensional. These 2D eigen spaces are parallel propagated. Thus a problem reduces to finding a parallel propagated basis in 2D spaces. They can be obtained from initially chosen basis by 2D rotations. The ODE for the angle of rotation can be solved by a separation of variables.



[Connell, V.F., Kubiznak, PRD 78, 024042 (2008)]

Possible generalizations to degenerate PCKY tensor and non-vacuum STs

Infinite set of new interesting completely integrable dynamical systems.

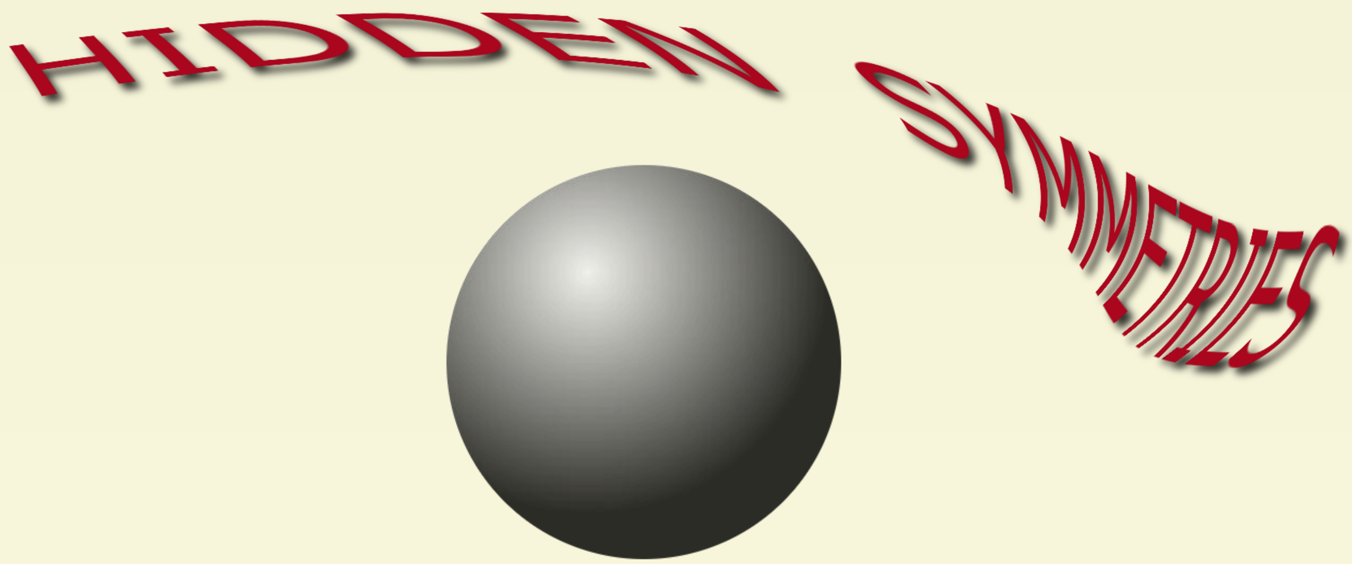
Lax-pairs for integrable geodesics in Kerr-NUT-(A)dS, (Cariglia, V.F., Krtous, Kubiznak, '13)

Deformed and twisted black holes with NUTs, (Krtous, Kubiznak, V.F., Kolar, '16)

Review in Living Review in Relativity (V.F., Krtous, Kubiznak), under preparation

A higher dimensional black hole demonstrates approximately half of its symmetries as explicit ST symmetries, while the other half is hidden.

BIG PICTURE



BLACK HOLES HIDE THEIR SYMMETRIES. WHY AT ALL HAVE THEY SOMETHING TO HIDE?

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`Alberta Separatists’