Interaction Energy of a Charged Medium and its EM Field in a Curved Spacetime

Mayeul Arminjon [1,2]

[1] *Lab. 3SR (Grenoble-Alpes University & CNRS), Grenoble, France.*

[2] *CNRS (Section of Theoretical Physics), France.*

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Motivation

Develop a consistent electrodynamics in an alternative theory of gravity: "Scalar ether theory" or SET.

Motivations for SET:

- Special relativity (SR) can be interpreted within classical concepts of space and time, thus keeping a "preferred" simultaneity: Lorentz-Poincaré interpretation/version of SR. SET extends this to gravitation. (Has curved spacetime too.)
- SET has a physical interpretation for gravity: a pressure force.
- Some problems of general relativity (GR) are avoided in SET: singularities (in gravitational collapse & cosmology), dark energy, interpretation of the gauge condition, a problem with Dirac equation.

PREVIOUS WORK

In GR, the eqs. of electrodynamics rewrite those of SR by using the "comma goes to semicolon" rule:
partial derivative $\rightarrow$ covariant derivative.

Not possible in SET, essentially because the Dynamical Equation isn't generally $T^{\lambda\nu}_{;\nu} = 0$ (which rewrites $T^{\lambda\nu}_{,\nu} = 0$ valid in SR).

In SET, first Maxwell group unchanged. Second group *was* got by applying the Dynamical Eqn of SET to a charged medium in the presence of Lorentz force, assuming (as is the case in GR) that

(i) Total energy-momentum tensor $\boldsymbol{T} = \boldsymbol{T}_{\text{charged medium}} + \boldsymbol{T}_{\text{field}}$.

(ii) Total energy-momentum tensor $\boldsymbol{T}$ obeys the Dynamical Eqn, without any non-gravitational force.

Previous work (more about it)

Assumptions (i) and (ii) lead to a form of Maxwell's second group in SET.

This form predicts charge production/destruction at unrealistic rates $\Rightarrow$ *discarded.* ($\exists$ also more theoretical reasons.)

Assumption (i) is contingent and may be abandoned. Means introducing "interaction" energy tensor $\boldsymbol{T}_{\text{inter}}$ such that

$$\boldsymbol{T}_{(\text{total})} = \boldsymbol{T}_{\text{charged medium}} + \boldsymbol{T}_{\text{field}} \underline{+ \boldsymbol{T}_{\text{inter}}} \,. \tag{1}$$

$\Rightarrow$ *Present work:* constrain the form of $\boldsymbol{T}_{\text{inter}}$ and derive eqs to calculate it in a realistic gravitational + EM field.
Needs attention to independent eqs & their number in electrodynamics. Let's begin with standard theory: SR and GR.

MAXWELL EQUATIONS IN STANDARD THEORY

Maxwell's *first group* for the EM antisymmetric field tensor $F_{\lambda\nu}$:

$$F_{\lambda\nu\,,\,\rho} + F_{\nu\rho\,,\,\lambda} + F_{\rho\lambda\,,\,\nu} \equiv F_{\lambda\nu\,;\,\rho} + F_{\nu\rho\,;\,\lambda} + F_{\rho\lambda\,;\,\nu} = 0 \qquad (2)$$

can be rewritten as

$$M_{\lambda\nu\rho} := F_{\lambda\nu\,;\,\rho} + F_{\nu\rho\,;\,\lambda} + F_{\rho\lambda\,;\,\nu} = 0, \qquad (3)$$

of which four eqs. are *linearly* independent, e.g.

$$M_{012} = 0, \quad M_{013} = 0, \quad M_{023} = 0, \quad M_{123} = 0. \qquad (4)$$

Maxwell's *second group* in SR and in GR: also 4 eqs:

$$F^{\lambda\nu}{}_{;\nu} = -\mu_0 J^\lambda \qquad (\lambda = 0, ..., 3). \qquad (5)$$

INDEPENDT EQS IN STANDARD THEORY (GIVEN SOURCE)

Assuming *source* J^λ *is given*, we thus have $4 + 4 = \mathbf{8}$ equations for **6** unknowns $F_{\lambda\nu}$ $(0 \leq \lambda < \nu \leq 3)$ (or **E** and **B**).

As is well known, those 8 eqs are nevertheless needed, e.g. $\mathrm{div}\,\mathbf{B} = 0$ can't be removed.

Can be explained by noting *two differential identities of the system:*

$$\text{I)} \quad e^{\lambda\nu\rho\sigma} M_{\lambda\nu\rho;\sigma} \equiv 0$$

for the first Maxwell group (3). For the 2nd group (5), we get first from $F^{\lambda\nu}{}_{;\nu;\lambda} \equiv 0$ charge conservation as a *compatibility condition:*

$$J^\lambda{}_{;\lambda} = 0. \tag{6}$$

If (6) is satisfied, then using again $F^{\lambda\nu}{}_{;\nu;\lambda} \equiv 0$ we get a differential identity for the 2nd group (5):

$$\text{II)} \quad \left(F^{\lambda\nu}{}_{;\nu} + \mu_0 J^\lambda\right)_{;\lambda} \equiv 0.$$

INDEPENDT EQS IN STANDARD THEORY (GENERAL CASE)

If the 4-current $\mathbf{J}$ is not given, we have at least $\mathbf{5}$ unknowns more: charge density of the charged continuum ρ_{el}, its 3-velocity field $\mathbf{v}$ ($\mathbf{J} \Leftrightarrow \rho_{\mathrm{el}}$ and $\mathbf{v}$), plus other state parameters of the continuum: say only its proper rest-mass density ρ^*.

Additional eqn: dynamical eqn for charged continuum (only $\mathbf{4}$ eqs):

$$T_{\mathrm{chg}\ \ ;\nu}^{\mu\nu} = F^{\mu}{}_{\lambda}\, J^{\lambda}. \tag{7}$$

(This implies the mass conservation at least for an isentropic fluid.)

However, now (II) is not a differential identity of the system any more: it applies only on the solution space. So $4+4+4-1=11$ independt eqs for $6+5=11$ unknowns.

INTERACTION ENERGY TENSOR IN SET

Recall: previous work showed that in SET we must consider

$$\boldsymbol{T} = \boldsymbol{T}_{\text{chg}} + \boldsymbol{T}_{\text{field}} \underline{+\boldsymbol{T}_{\text{inter}}} \neq \boldsymbol{T}_{\text{chg}} + \boldsymbol{T}_{\text{field}} \tag{8}$$

in the dynamical equation. (Here chg = charged medium.) The latter coincides with $T^{\mu\nu}_{;\nu} = 0$ in a constant gravitational field, in particular in the "situation of SR" (SET with metric = Minkowski's). Now in SR (as well as in GR) we have:

$$T^{\mu\nu}_{\text{chg}\ ;\nu} + T^{\mu\nu}_{\text{field}\ ;\nu} = 0. \tag{9}$$

Therefore, in the situation of SR, we should have in SET:

$$T^{\mu\nu}_{\text{inter}\ ;\nu} = 0. \tag{10}$$

Dynamical equations in SET

Dynamical equation for the total energy tensor in SET:

$$T^{\mu\nu}_{;\nu} = b^{\mu}(\mathbf{T}), \tag{11}$$

where

$$b^0(\mathbf{T}) := \frac{\gamma^{00}}{2} g_{ij,0}\, T^{ij}, \quad b^i(\mathbf{T}) := \frac{1}{2} g^{ij} g_{jk,0}\, T^{0k}, \tag{12}$$

with γ the spacetime metric, and where $\boldsymbol{g}$ is the spatial metric in the preferred reference frame $\mathcal{E}$ assumed by SET.

Dynamical equations in SET (continued)

For a continuous medium in the presence of a field of external non-gravitational 3-force $\mathbf{f} = (f^i)$ $(i = 1, 2, 3)$:

$$T^{\mu\nu}_{\text{medium} \ ;\nu} = b^\mu(\boldsymbol{T}_{\text{medium}}) + f^\mu, \qquad f^0 := \frac{\mathbf{f.v}}{c\beta}, \tag{13}$$

where $\beta := \sqrt{\gamma_{00}}$ and $\mathbf{v}$ is the 3-velocity field defined with the local time.

For a charged medium ($\boldsymbol{T}_{\text{medium}} = \boldsymbol{T}_{\text{chg}}$) subjected to EM field, we get $f^\mu = F^\mu_{\ \nu} J^\nu$, so (13) is

$$T^{\mu\nu}_{\text{chg} \ ;\nu} = b^\mu(\boldsymbol{T}_{\text{chg}}) + F^\mu_{\ \nu} J^\nu. \tag{14}$$

Independent Eqs and Unknowns for SET

Independent eqs: same structure as in GR:

- Maxwell's first group (3): 4
- Dynamical eqn for the total energy tensor (11): 4
- Dynamical eqn for the charged medium (14): 4
- minus one differential identity $e^{\lambda\nu\rho\sigma} M_{\lambda\nu\rho;\sigma} \equiv 0$: -1

So 11 independent equations.

Independent unknowns also close to GR:

- EM field $F_{\mu\nu}$ $(0 \leq \mu < \nu \leq 3)$: 6
- 4-current $\mathbf{J}$: 4
- proper rest-mass density ρ^*: 1
- *plus* at least one new field to define $\boldsymbol{T}_{\text{inter}}$ ≥ 1

So ≥ 12 independent unknowns. ≥ 1 equation more is needed.

DYNAMICAL EQN WITH ENERGY INTERACTION TENSOR: I

We use the general decomposition of the total energy tensor $\boldsymbol{T}$ (8). Then the dynamical equation (11) for $\boldsymbol{T}$ in SET is equivalent to:

$$T^{\mu\nu}_{\text{field}\ \ ;\nu} = b^\mu(\boldsymbol{T}_{\text{field}}) + b^\mu(\boldsymbol{T}_{\text{chg}}) - T^{\mu\nu}_{\text{chg}\ \ ;\nu} + b^\mu(\boldsymbol{T}_{\text{inter}}) - T^{\mu\nu}_{\text{inter}\ \ ;\nu}. \tag{15}$$

Maxwell's 1st group implies an identity for the energy tensor of the EM field:

$$\mu_0 T^{\mu\nu}_{\text{field}\ \ ;\nu} \equiv F^\mu_{\ \ \lambda} F^{\lambda\nu}_{\ \ \ ;\nu}. \tag{16}$$

By using this and the dynamical equation (14) for the charged medium, (15) rewrites as

$$F^\mu_{\ \ \lambda} F^{\lambda\nu}_{\ \ \ ;\nu} = \mu_0 \left[b^\mu(\boldsymbol{T}_{\text{field}}) - F^\mu_{\ \ \nu} J^\nu - \delta^\mu \right], \tag{17}$$

where

$$\delta_\mu := T_{\text{inter}\ \mu\ \ ;\nu}^{\ \ \nu} - b_\mu(\boldsymbol{T}_{\text{inter}}). \tag{18}$$

DYNAMICAL EQN WITH ENERGY INTERACTION TENSOR: II

If the matrix $(F^{\mu}{}_{\lambda})$ is invertible ($\Leftrightarrow$ $\mathbf{E}.\mathbf{B} \neq 0$), (17) becomes

$$F^{\mu\nu}{}_{;\nu} = \mu_0 \left[G^{\mu}{}_{\nu} \left(b^{\nu}(\boldsymbol{T}_{\text{field}}) - \delta^{\nu} \right) - J^{\mu} \right], \tag{19}$$

with $(G^{\mu}{}_{\nu}) := (F^{\mu}{}_{\nu})^{-1}$.

Using $F^{\lambda\nu}{}_{;\nu;\lambda} \equiv 0$ gives

$$J^{\mu}{}_{;\mu} = \left[G^{\mu}{}_{\nu} \left(b^{\nu}(\boldsymbol{T}_{\text{field}}) - \delta^{\nu} \right) \right]_{;\mu}. \tag{20}$$

Lorentz-invariant interaction energy

In SR the interaction energy tensor $\boldsymbol{T}_{\text{inter}} = \mathbf{0}$. In SET we may impose that, without gravitational field, $\boldsymbol{T}_{\text{inter}}$ should be Lorentz-invariant. This is true iff we have when the metric γ is Minkowski's ($\gamma_{\mu\nu} = \eta_{\mu\nu}$ in Cartesian coordinates):

$$T_{\text{inter}\ \mu\nu} = p\,\eta_{\mu\nu} \qquad \text{(situation of SR)}, \tag{21}$$

with some scalar field p. This is equivalent to:

$$T^{\mu}_{\text{inter}\ \ \nu} := \eta^{\mu\rho}\,p\,\eta_{\rho\nu} = p\,\delta^{\mu}_{\nu} \qquad \text{(situation of SR)}. \tag{22}$$

The definition

$$T^{\mu}_{\text{inter}\ \ \nu} := p\,\delta^{\mu}_{\nu}, \qquad \text{or} \quad (T_{\text{inter}})_{\mu\nu} := p\,\gamma_{\mu\nu}, \tag{23}$$

thus got in Cartesian coordinates in a Minkowski spacetime, is actually generally-covariant. Therefore, we adopt (23) for the general case.

Charge conservation in SET

$\diamondsuit$ With the "scalar" interaction energy tensor (23) we have just one unknown more: the field p.

We may add the charge conservation as the new scalar eqn.

In view of (20), this determines the field p in a given gravitational + EM field.

$\diamondsuit$ In contrast, when in previous works we assumed the usual additivity of energy tensors, i.e., $\boldsymbol{T}_{\text{inter}} = \mathbf{0}$, then the system of eqs of electrodynamics of SET was closed.

Thus in the latter case, charge conservation could'nt be imposed. In fact we then got significant charge production/destruction...

WEAK GRAVITATIONAL FIELD

An asymptotic framework was developed for an EM field in a weak and slowly varying grav. field (e.g. talk/paper at 2017 GIQ Conf.). Essentially: conceptually associate with given system S a *family* (S_λ) of systems, depending on $\lambda \to 0$, $\lambda = 1/c^2$ in specific λ-dependent units. Of course the EM field is *not* assumed weak nor slowly varying. Write Taylor expansions w.r.t. λ: e.g.

$$\boldsymbol{F} = c^n \left(\overset{0}{\boldsymbol{F}} + c^{-2} \overset{1}{\boldsymbol{F}} + O(c^{-4}) \right) \tag{24}$$

where n could be any integer. And

$$p = c^{2n-5} \left(\overset{0}{p} + c^{-2} \overset{1}{p} + O(c^{-4}) \right), \tag{25}$$

for the "interaction scalar" field p with $\boldsymbol{T}_{\mathrm{inter}} = p\,\gamma$, Eq. (23). (The order $2n-5$ follows from charge conservation with (20).)

INTERACTION SCALAR IN A WEAK GRAVITATIONAL FIELD

Writing charge conservation with (20), and using such asymptotic expansions, we get for the first-approximation field $p_1 := c^{2n-5} \overset{0}{p}$:

$$\partial_T \, p_1 + u^j \partial_j \, p_1 = S, \tag{26}$$

where

$$S := \frac{c^{-2} \left(e^i \partial_T U\right)_{,i}}{k^0} \tag{27}$$

and

$$u^j := \frac{c \, k^j}{k^0}. \tag{28}$$

Here U is the Newtonian grav. potential, while the e^i 's and k^μ 's depend only on the first-approximation EM field $(\mathbf{E}, \mathbf{B})$, that obeys the standard flat-spacetime Maxwell equations.

Advection equation for the interaction scalar

Equation (26) is an advection equation with a given source S for the unknown field p_1. This is a hyperbolic PDE whose characteristic curves are the integral curves of the vector field $\mathbf{u} := (u^j)$.

That is, on the curve $\mathcal{C}(T_0, \mathbf{x}_0)$ defined by

$$\frac{\mathrm{d}\,\mathbf{x}}{\mathrm{d}\,T} = \mathbf{u}(T, \mathbf{x}), \qquad \mathbf{x}(T_0) = \mathbf{x}_0, \tag{29}$$

we have from (26):

$$\frac{\mathrm{d}\,p_1}{\mathrm{d}\,T} = \frac{\partial p_1}{\partial T} + \frac{\partial p_1}{\partial x^j}\frac{\mathrm{d}\,x^j}{\mathrm{d}\,T} = S(T, \mathbf{x}). \tag{30}$$

Solution of the advection equation

We note that the field $\mathbf{u}$ is *given*, i.e. it does not depend on the unknown field p_1: Eq. (28).

Therefore, the integral lines (29) are given, too, hence the characteristic curves do not cross. Thus, the solution p_1 is got uniquely by integrating (30):

$$p_1(T, \mathbf{x}(T)) - p_1(T_0, \mathbf{x}_0) = \int_{T_0}^{T} S(t, \mathbf{x}(t)) \, \mathrm{d}\, t, \qquad (31)$$

where $T \mapsto \mathbf{x}(T)$ is the solution of (29).

If at time T_0 the position $\mathbf{x}_0$ in the frame $\mathcal{E}$ is enough distant from material bodies, one may assume that $p_1(T_0, \mathbf{x}_0) = 0$.

Conclusion

— Differential identities show that in standard electrodynamics: number of independent scalar PDE's = number of unknowns. True with given 4-current $\mathbf{J}$ and also with $\mathbf{J} \in \{\text{unknowns}\}$.

— Same is true in the investigated theory of gravity ("SET") with additivity assumption ($\boldsymbol{T}_{\text{inter}} = \mathbf{0}$). That however leads to charge non-conservation. Thus one must have in general $\boldsymbol{T}_{\text{inter}} \neq \mathbf{0}$.

— For $\boldsymbol{T}_{\text{inter}}$ to be Lorentz-invariant in SR, it must involve just a scalar field p. Then need one more equation: charge conservation.

— In a weak and slowly varying gravitational field and with a given EM field, the scalar field p is determined by an advection equation with given source, Eq. (26). Hence p may be calculated by integration along characteristics, Eq. (31). The corresponding interaction energy could be counted as "dark matter", since it is not especially localized inside matter.