Geometric Algebra for Applications in Cybernetics: Image Processing, Neural Networks, Robotics and Integral Transforms

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ABSTRACT

Motivation

Geometric Algebra (GA) achieves the unification of many domains like multi linear algebra, multivariable analysis, conformalgeometry. Lie group and Lie algebras, projective geometry, quantum mechanics and general relativity. GA allows:

- Extending and completing algebraic operations on vectors and multi-vectors.
- Unified concept for geometry and algebra
- Explicit interpretation of geometric entities and operations on them in arbitrary dimensions with Euclidean and pseudo-Euclidean metrics.
- Advanced formalism for rotations and in general Lie groups in arbitrary dimensions.
- Metalanguage for high level reasoning

Geometric Algebra has a wide range of applications: graphics engineering, signal and image processing, computer vision, neural networks and quantum computing, machine learning and robotics.

Cybernetics and Geometric Algebra

Cybernetics encompasses a variety of diverse fields, where the common characteristic is the existence of a perception action cycle system with feedback and a controller, so that the system adapts itself and ultimately augments its degree of awareness and autonomy to interact and maneuver successfully in its environment. The modeling of the systems and the processing of entities is crucial for an efficient system behavior; according Fourier and Chasles, it is more advantageous to model

and process data in representations in higher dimensionality pseudo Euclidean spaces with a nonlinear computational framework like the horosphere. Geometric entities like lines,planes, circles, hyper planes and spheres can be used for screw theory in kinematics and dynamics, geometric neural computing and the design of observers and controllers for perception action systems. David Hestenes has contributed to developing geometric algebra as a unifying language for mathematics and physics [1,2,3] and Eduardo Bayro-Corrochano [4] for perception and action systems. Due to the versatility and the mathematical power of the geometric algebra framework, it can be of great use for handling diverse and complex problems in artificial intelligence, robotics, computer vision, second order cybernetics, self-organization in cybernetics, geometric neural networks, Clifford Fourier and wavelet transforms, fuzzy geometric reasoning, cognitive architectures, quantum computing and robots and humanoids. Geometric algebra constitutes undoubtedly an appropriate mathematical system for cybernetics.

Lecture 1
Introduction to Geometric Algebra
Summary

Here we explain the basic geometric and algebraic concepts of Geometric Algebra.

1.1 Basic definitions

1.2 Multi vector products

1.3 Further properties of the geometric product

1.4 Dual blades and duality in the geometric product

1.5 Multi vector operations

1.6 The vector derivative

1.7 Grad, div and curl

1.8 Applications

Lecture 2
2D and 3D Geometric Algebras
Summary

In this second lecture we will introduce the geometric algebras of the 2D and 3D space, and study some of their relevant features. This will enable us to build up a picture of how geometric algebra can be employed to solve interesting problems in engineering and computer science.

2.1 Complex, double and dual numbers

2.2 Geometric algebra of the plane

2.3 Geometric algebra for the 3D Euclidean space

2.4 Quaternions, Rotors and Motors

2.5 Lie groups using Bivector algebras

2.6 Rigid body spatial velocity using rotor algebra

2.7 Applications

Lecture 3
Summary

In the framework of rotor and motor algebra, we will model the kinematics, differential kinematics and dynamics of mechanical systems using the basic geometric entities point, lines and plane and show the fundaments for interesting applications in graphics engineering, interpolation and robotics.

3.1 Motor Algebra

3.2 Kinematics of the 3D and 4D Spaces

3.3 Applications

Lecture 4
Summary

To gain understanding and be trained in algebraic manipulation of GA equations, it is necessary to study projective geometry which is needed to understand high dimensional geometric algebras like the conformal geometric algebra. This lecture is devoted to study the projective geometry of the projective plane and projective space which are essential for applications in computer vision, graphic engineering and interpolation.

4.1 Projective Geometry and Algebra of Incidence

4.2 Lie Groups of Projective Geometry

4.3 Applications

Lecture 5

Summary

This fifth lecture offers an introduction to Conformal Geometric Algebra (CGA). The computational entity of this framework is the sphere which helps for modeling other geometric primitives as points, lines and planes and ruled surfaces as well. For instance,in a robust digital camera calibration, one can get rid of the involved affine transformation.We will show that CGA can be used in its full extend in robotics for closing the gap between the visual and mechanical world representations.

5.1 Null Cone, Hyperplane and Horosphere

5.2 Sphere, Line, Plane, Pair of Points

5.3 Duality, Meet and Join

5.4 Conformal Transformation

5.5 Versors and Lie Groups

5.6 Applications

References

[1] David Hestenes, *Space-Time Algebra*, Gordon and Breach, New York 1966.

[2] David Hestenes and Garret Sobczyk, *Clifford Algebra to Geometric Calculus - A Unified Language for Mathematical Physics*, Reidel, Dordrecht 1984.

[3] David Hestenes, *New Foundations for Classical Mechanics*, Reidel, Dordrecht 1986.

[4] Eduardo Bayro-Corrochano, *Geometric Computing for Wavelet Transforms. Robot Vision, Learning, Control and Action*, Springer, London 2010.