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Book Reviews

Edited by R. B. Kellogg

Featured Review: Recent Books on Mathematical Finance

Introduction to Stochastic Calculus Applied to Finance. *By D. Lamberton and B. Lapeyre.* (Translated by N. Rabeau and F. Mantion.) Chapman & Hall, London, 1996. $64.95. xi + 185 pp., hardcover. ISBN 0-412-71800-6.

Introduction to Mathematical Finance: Discrete Time Models. *By Stanley R. Pliska.* Blackwell, Malden, MA, 1997. $52.95. x + 262 pp., hardcover. ISBN 1-55786-945-6.

Financial Calculus: An Introduction to Derivative Pricing. *By Martin Baxter and Andrew Rennie.* Cambridge University Press, Cambridge, UK, 1996 (reprinted with corrections, 1997, 1998). $42.95. ix + 233 pp., hardcover. ISBN 0-521-55289-3.

Generalised Optimal Stopping Problems and Financial Markets. *By Dennis Wong.* Addison Wesley Longman, Ltd., Essex, UK, 1996. $40.33. vi + 114 pp., softcover. Pitman Research Notes in Mathematics Series, Vol. 358. ISBN 0-582-30400-8.

American Put Options. *By D. M. Salopek.* Addison Wesley Longman, Ltd., Essex, UK, 1997. $56.46. 113 pp., hardcover. Pitman Monographs and Surveys in Pure and Applied Mathematics, Vol. 84. ISBN 0-582-31594-8.

Exotic Options: A Guide to Second Generation Options. *By Peter G. Zhang.* World Scientific, Singapore, 1997. $96.00. xvii + 675 pp., hardcover. ISBN 981-02-2222-X.

Martingale Methods in Financial Modelling. *By Marek Musiela and Marek Rutkowski.* Springer-Verlag, Berlin, 1997. $79.95. xii + 512 pp., hardcover. Applications of Mathematics, Vol. 36. ISBN 3-540-61477-X.

1. Overview. Mathematical finance is a new subject and a fascinating one. It uses recently developed mathematics in immediate applications where large sums of money are involved. Because of its applicability, there is a strong demand from the investment community for highly trained and competent technicians. At the same time, finance theory has created a significant research thrust in probability theory from a new direction, creating new problems, new ways of viewing traditional objects, and new theories.

Before discussing the books under review, let us situate the emerging literature. A probabilistic analysis of the stock market began in 1900 with the thesis of a young French mathematician, Louis Bachelier. Both subjects, economics and probability, were under shadows of suspicion in the world of French mathematics at the time, but the latter was more acceptable, as it was motivated by physics. Bachelier essentially invented Brownian motion five years before Einstein's famous 1905 paper and decades before Kolmogorov gave mathematical legitimacy to the 300-year-old subject of probability theory. Bachelier's thesis was not well received and, as a result, he was blackballed by the mathematical establishment and banished to internal exile at Besançon, a small provincial capital in the mountains near Switzerland. Indeed,

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~~out of bounds. The user should insert stopping criteria to prevent these occurrences.~~

~~In summary, Acheson's book is an attractive and clear presentation of the evolution of ideas in nonlinear dynamics. Mathematicians, physicists, and engineers from all levels will find it rewarding and an entertaining exposition of a semitechnical nature.~~

~~J. David Logan~~
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Algebraic Methods in Quantum Chemistry and Physics. *By Francisco M. Fernandez and Eduardo A. Castro.* CRC Press, Boca Raton, FL, 1996. $110.00. vii+269 pp., hardback. ISBN 0-8493-8292-0.

This is a book on applications of Lie algebras and operator methods in theoretical physics and chemistry, and like any other book on subjects as vast as these, it has a point of view that guided the selection of topics. The authors' point of view is to omit the exposure of basic notions and theorems in the theory of Lie groups and Lie algebras, assuming that the reader is familiar with the main ideas and concepts relevant to those fields, and, instead, to provide some useful mathematical tools needed for derivation of the theoretical results in a most economic and elegant way. In particular, the Campbell–Baker–Hausdorff formula, disentangling of the exponential operators, and Liouville transformation receive such a treatment. At the same time, this book can serve as a complement to the standard quantum-mechanical course, because most of the fundamental notions, like observables, representations, eigenvectors, eigenvalues, matrix elements, selection rules, coherent states, equations of motion, approximation and perturbation methods, and the Schrödinger, Heisenberg, and intermediate pictures in quantum mechanics are introduced, explained, and exemplified. It should be stressed that quantum mechanics per se is a well-established discipline, and the actual question that arises is: How should one solve the concrete problem?

Let us also point out that quantum-mechanical problems capable of exact solution are not numerous and are traditionally solved in the textbooks on the subject mainly by means of Schrödinger's wave equation. On the contrary, the operator methods are usually applied only in a few instances, like harmonic oscillator and angular momentum problems. The present book shows that a large number of one- and some three-dimensional models are solvable by algebraic, representation-independent methods relying only on the standard commutation relations. In the more realistic situations, it is shown how one can enlarge the above approach by making use of various combinations of ladder or shift operators methods, of the virial and hypervirial theorems, and sometimes of computer calculations in order to find the appropriate solution. Applications of all these methods to the calculation of eigenvalues, matrix elements, and wave functions are discussed in detail and, in many cases, up to numerical evaluations. The concrete physical problems that receive such treatment include the computation of the vibrational overlap integrals (Franck–Condon factors), some exactly solvable models with spherical symmetry, and the vibration-rotational spectrum of diatomic molecules, just to mention a few. The more mathematically inclined reader will be pleased to find a clear exposition (besides the above-mentioned Campbell–Baker–Hausdorff formula) of the Magnus theorem, the Fokker–Planck equation, the Wigner–Kirkwood expansion, and the Euler–MacLaurin and Poisson summation formulae.

In short, this book is a nice introduction to the use of operator methods in quantum mechanics and chemistry and can also serve as a reference source because of the numerous problems solved in it. Without any doubt, it is quite suitable for use by students in intermediate quantum mechanics courses and also by more advanced postgraduate students and researchers who wish to see how efficient the algebraic methods are in solving concrete quantum-mechanical problems. The experienced reader can also consult some of the relevant competitive books on the subject (and not cited in this book), which are listed below in order of their appearance.

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Geometrical Methods in Robotics. *By J. M. Selig.* Springer-Verlag, New York, 1996. $29.95. xii+269 pp., hardback. ISBN 0-387-94728-0.

The book under review aims to show the power and elegance of Lie groups and some algebraic and geometric concepts relevant to the problems of robotics. At the beginning, some basic notions from algebraic and differential geometries are given and an exposition of the theory of Lie groups is presented, namely, definitions and examples including: matrix groups, homomorphisms, actions and products, and the main properties of the Euclidean group. Then the theory of subgroups is given, and especially, the notion of a Lie subgroup is discussed in some depth. After introducing quotients and normal subgroups, the subgroups of SE(3) receive special attention here. This is justified at least by considerations related to the introduction and use of the notion of the Reuleaux pair, which is recognized as a surface in the three-dimensional space that is invariant under the action of a subgroup of SE(3). In this way, all possible Reuleaux pairs that are found to be the orbits of all possible subgroups are investigated. After that the forward and inverse kinematic problems are formulated. Chapter 4 studies some basic elements from the theory of Lie algebras. Lie algebras are described as the tangent spaces to the identity elements, and the tangent vectors to a general manifold are defined here as well. Then follow the definitions of commutators and the adjoint representation of the group. One can look at Lie algebra elements as left- or right-invariant vector fields on the group and this is pursued in the book as well. Another important construction is the exponential map, which by definition is a mapping from the Lie algebra to the group and has been used for the purposes of robot kinematics which is written in exponential form. As an example, the Rodrigues formula is derived while the theory is applied to finding jacobians that appear in any real treatment of robotic problems. Besides, the Killing form and the Campbell Baker Hausdorff formula are defined and explained. After this overview of the theoretical background, Selig directs his attention to special problems of robotics. Chapter 5 considers the inverse kinematic problem for $3-R$ wrists, $3-R$ robots, and kinematics of planar motion. The Euler Savaray relation is defined and the equation of the inflection circle is obtained. The point called undulation is introduced along with the special Ball's point. Further on, notions about the cubic of stationary curvature and the Brumester points are discussed. The above theory is applied to the planar, four-bar mechanism. Next, in Chapter 6, the author treats the subject of line geometry. Some of the classical theory of ruled surfaces and line complexes is introduced from the Lie algebra point of view. For robotics, the most important ruled surfaces are cylindrical hyperboloids and cylindroids. A full description of these surfaces is given. The theory of group representations is introduced in Chapter 7, and again the emphasis is on the group of proper Euclidean motions. Relying on them, the classical and a modern statement of the "Principle of Transference" is formulated. The infinitesimal screws of Ball can be seen