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BOOK REVIEWS

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Selected Topics in Approximation and Computation. *By Marek A. Kowalski, Krzysztof A. Sikorski, and Frank Stenger.* Oxford University Press, Oxford, UK, 1995. $65.00. xiv + 349 pp., hardcover. ISBN 0-19-508059-9.

Weierstrass and Tchebycheff laid the foundations of the theory of approximations by giving two beautiful results: (i) every continuous function is a limit of a sequence of polynomials, (ii) the polynomial of best uniform approximation to a given continuous function is completely characterized by an equioscillation property. After more than a century of intensive research this theory now covers a vast area of analysis and applications. Well-motivated problems from the practice and interesting questions from the intrinsic logic of the development of the theory itself have been keeping alive the activity and the vitality of this mathematical subject. Only in recent years have several waves of intensive investigations thrown the borders of the theory of approximation far ahead and into neighboring fields. Some of these new topics include n-widths, optimal algorithms, spline functions, wavelets, and computer-aided geometric design. Books devoted to all of these particular domains have been published. Consequently, it is a difficult job to write a single book covering the main achievements in approximation theory. The authors of the book under review wisely chose the title warning that they will touch only some selected topics from this wide and still very active domain. As is easy to guess, most of the topics chosen are those the authors have contributed to.

Chapter 1 is devoted to classical questions: approximation in normed spaces, characterization of the best approximation, Fourier series, orthogonal polynomials, Korovkin operators, direct and inverse theorems, and projection operators. This part, based on the best texts ever written, is very well presented and, if supplemented with some other important results, could be used as an introductory course on approximation theory. The next chapter deals with splines. In 25 pages the authors introduce the beginner to the subject. Some basic definitions and facts concerning mainly interpolation by natural cubic splines and properties of the B-splines are given. Next, Chapters 3 and 4 are devoted to sinc approximations and sinc algorithms. This is a domain in which one of the authors (Frank Stenger) is an expert. His 550 page monograph on sinc functions appeared in 1993, published by Springer. Theoretical background and basic tools in the sinc approximation are presented in Chapter 3 while Chapter 4 contains explicit computational methods. This is a concise introductory text to modern and powerful approximation techniques. Chapter 5 (Moment Problems) and Chapter 6 (n-widths and s-numbers), as the authors stress, "barely scratch the surface of the subject." Chapter 7 is devoted to optimal approximation methods. This is a relatively new development in approximation theory and numerical analysis. The main concept here is to construct an approximation scheme based on partial information that has a minimal error in a given class of functions. Finally, the book ends with a short chapter containing some applications: the sinc solution of Burger's equation, the best approximation of band-limited signals, and the bisection method for zero finding.

All sections end with exercises.

Some of the topics are very well presented, incorporating elegant and simple proofs.

The book gives an easy-to-read introduction to classical and modern questions in

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~~Consider the control problem~~

$$\ddot{\psi} - \Delta\psi = 0 \quad \text{in } \Omega \times (0,T),$$
$$\frac{\partial\psi}{\partial\nu} = u \quad \text{on } \Gamma \times (0,T),$$
$$\psi|_{t=0} = \psi^0, \quad \dot{\psi}|_{t=0} = \psi^1 \quad \text{in } \Omega.$$

~~A short computation shows that~~

$$E(T) = E(0) + \int_0^T \int_\Gamma u\dot{\psi}\, d\Gamma dt, \quad T > 0.$$

~~If, therefore, one chooses $u = -k\dot{\psi}$, where $k > 0$, then $E(T) \leq E(0)$; that is, the system is dissipative. The stabilization problem involves showing that $E(t) \to 0$ as $t \to \infty$ (called strong asymptotic stability) or establishing the more demanding conclusion $E(t) \leq Ce^{-\omega t}E(0)$ for some $\omega > 0$ (called uniform asymptotic stability). The latter is equivalent to proving the stability estimate~~

$$E(T) \leq C_T \int_0^T \int_\Gamma \left(\frac{\partial\psi}{\partial\nu}\right)^2 d\Gamma dt, \quad T > T_0$$

~~for some $T_0 > 0$. Again, such an estimate is amenable to multiplier methods of the type discussed above, although certain other ideas are also involved. One may consider nonlinear dissipative feedback control laws such as $u = -f(\dot{\psi})$ as well, where f is a monotone increasing function with $f(0) = 0$, and obtain asymptotic estimates of $E(t)$ by multiplier methods.~~

~~With the above background, one may say that the main point of the book under review is to obtain hidden regularity estimates, observability estimates, and/or stabilization estimates for various evolutionary boundary value problems, by the use of appropriate multipliers, and to interpret such estimates in the context of exact controllability and/or asymptotic stability. The author is a master of the multiplier method and the estimates presented are typically the sharpest obtainable by this conceptually simple, yet powerful, technique. The book is organized into 11 brief chapters. Chapter 0 provides a simple paradigm, where all computations are done explicitly, in the form of a vibrating string, controlled at its endpoints. Background material on second-order linear evolutionary equations in a Hilbert space is presented in Chapter 1, and the three prototypical boundary value problems that are considered throughout the book are introduced there. The first of these is the wave equation with a potential and with mixed Dirichlet and Robin boundary conditions. The second and third concern the equation $\ddot{\phi} + \Delta^2\phi = 0$, which arises in thin plate theory, with either Dirichlet boundary conditions or boundary conditions $u = \Delta u = 0$. Chapter 2 is devoted to the issue of hidden regularity for these three systems, while in Chapter 3 observability estimates are obtained for the same systems. The implications of the results of Chapter 3 for exact controllability are delineated in Chapter 4. The results of Chapter 4 are refined in Chapter 6 by using some nice results on norm inequalities that are presented in Chapter 5. Chapters 7 through 9 are devoted to questions of asymptotic stability, by means of various dissipative feedback controls at the boundary, in the context of the wave equation with a potential, Maxwell's equations, and the fourth-order equation mentioned above. The final chapter treats asymptotic stability of the Korteweg de Vries equation~~

$$\dot{u} + uu_x + u_{xxx} = -k(u - [u])$$

~~on the interval $0 \leq x \leq 1$, with periodic boundary conditions, where $[u]$ denotes the spatial average value of u. The main result is that in a certain topology u behaves asymptotically like the average value of the initial state.~~

~~This book provides a highly readable entrée into the enormous literature on exact controllability and stabilizability of evolutionary equations that has appeared in the last 15 or so years, together with an extensive bibliography to guide those who wish to delve further into these matters.~~

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Group Theory and Physics. *By S. Sternberg.* Cambridge University Press, Cambridge, UK, 1995. $29.95. xiii + 429 pp., paper. ISBN 0-521-55885-9.

The usual starting point of many books devoted to applications of group theory to physics is the exposure of its basic notions and theorems. For most readers this is quite a serious psychological barrier as the necessity and naturality of these notions and theorems are clarified after transition to applications. The author of this book takes another course, and in its first chapter introduces to the reader plenty of examples which illustrate the fundamental notions of "symmetry" and "symmetry of the problem." The three main sources of symmetries in the real physical problems can be roughly classified as follows:

(i) the symmetries of space and time,

(ii) indistinguishability of the elementary particles of the same kind,

(iii) symmetries of various model physical systems.

The space–time symmetries manifest themselves by an equivalence of all inertial coordinate systems. In plain words this means that the physical laws are formulated identically in all inertial systems. As an example one can consider Maxwell equations. In his epoch-making paper Einstein proved that these equations are invariant with respect to Lorentz transformations of space and time coordinates. The set of all such space–time transformations form a ten-dimensional Lie group which Wigner named the Poincare group, and what is really important—the physical quantities undergo linear transformations as well. This is the way in which the classification of the representations of the Poincare group enters into physical considerations. More common in practice are its subgroups of rotations of the ordinary three-dimensional space and proper Lorentz transformations. This results in a classification of the physical quantities according to their representations. Just in this way scalars, vectors, and other tensors of different ranks arise. Further, quantum mechanics reveals the fundamental role which the covering group of rotations plays, and this leads to the appearance of new physical objects known as spinors of various ranks.

The second origin of symmetries in the physical problems—the indistinguishability of the elementary particles of the same kind—has the effect that any permutation of two or more identical particles is obviously a symmetry operation for the system under consideration. Besides, the theory of the permutations groups developed mainly by Frobenius and Schur provides the model for discussion of more complex groups.

Finally, the third occurrence of the powerful group methods is in the search for higher symmetries in the imaginary worlds. As an example, the electromagnetic interaction of the protons in the nucleus can be neglected so that the neutrons and protons to be treated in main as different states of nucleons.

All these aspects of symmetry manifestations are discussed in depth in Sternberg's book. The first chapter starts with basic definitions and examples and ends with a detailed treatment of the fullerene molecule. The second chapter provides the basic facts from representation theory of the finite groups. Next comes Chapter three, in which molecular vibration is explained using vector bundles methodology. The induced representations method of Frobenius developed here culminates in Mackey theory. Compact and some noncompact Lie groups which play major roles in physics are discussed more systematically in the fourth chapter. The final chapter is devoted to representation theory of the unitary groups. All this takes approximately 300 pages, which are supplemented by another approximately hundred pages in the form of seven appendices. They comprise more technical aspects and rigorous proofs of the basic theorems in representation theory omitted in the main text. The only exception is Appendix F, in which the history of 19th century spectroscopy is presented.

As a whole, this is a book written by a mathematician for a mathematical audience and could serve ideally as a text for an advanced graduate course. As with other Sternberg writings, the subject is well shaped and the exposition is fairly lucid.

In conclusion I consider Sternberg's book a fine addition to the existing literature and strongly recommend it to anyone with an interest in learning how to use group-theoretical methods to understand concrete physical problems. The potential reader might wish to also have a look at some "physical books." A noncomprehensive list of titles which belong in this category can be found below.

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~~**Elliptic Marching Methods and Domain Decomposition.** *By Patrick J. Roache.* CRC Press, Boca Raton, FL, 1995. $69.95. 190 pp., hardcover. ISBN 0-8493-7378-6.~~

~~This book examines direct marching methods for the numerical solution of general elliptic problems. These methods are essentially unstable, imposing a limitation on the problem size. However, there are ways to work around this constraint and the resulting efficient algorithms have powerful applications to important problems in engineering. They lead to a class of iterative methods for linear problems with very rapid convergence and to the solution of nonlinear steady state problems by semidirect methods.~~

~~The treatment is highly computational with a detailed description of operation counts, storage requirements, and timing tests. The material is presented in a clear and concise way. Stability results are mostly empirical, based on the extensive numerical experience of the author.~~

~~The book consists of nine chapters and three appendices. Chapter 1 discusses marching methods for elliptic problems in one and two dimensions and introduces the basic ideas that make the methods usable by controlling their inherent instability. For second-order elliptic equations, a transparent explanation of the application of marching methods to irregular boundary geometries, advection diffusion equations, upwind differences for advection terms, turbulence terms, cross derivatives, Helmholtz terms, interior flux boundaries, and others is given. Chapter 2 is devoted to higher-order equations, both in the sense of higher-order accuracy solutions to second-order equations and in the sense of higher-order elliptic equations. The direct marching method is utilized here in combination with rapidly converging iterative schemes. In Chapter 3 several direct and iterative methods for extending the mesh size and controlling stability are described. These lead naturally to the implementation of domain decomposition techniques for large problems. Special approximations to the influence coefficient matrix, obtaining significant savings in operation counts and storage, are developed in Chapter 4. Chapter 5 briefly introduces marching methods for solving three-dimensional elliptic problems. In Chapter 6 the performance of two-dimensional general elliptic marching codes is evaluated through a variety of timing and accuracy tests. Chapter 7 discusses the vec-~~