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Book Reviews

Edited by R. B. Kellogg

Featured Review: Selected Books on Numerical Linear Algebra

Numerical Linear Algebra and Applications. *By Biswa Nath Datta.* Brooks/Cole, Pacific Grove, CA, 1995. $83.95. xxii+680 pp., hardcover. ISBN 0-534-17466-3.

Applied Numerical Linear Algebra. *By James W. Demmel.* SIAM, Philadelphia, PA, 1997. $45.00. xi+419 pp., softcover. ISBN 0-89871-389-7.

Numerical Linear Algebra. *By Lloyd N. Trefethen and David Bau* III. SIAM, Philadelphia, PA, 1997. $34.50. xii+361 pp., softcover. ISBN 0-89871-361-7.

The world of numerical linear algebra has always been well endowed with outstanding books. Wilkinson's classic monograph *The Algebraic Eigenvalue Problem* [13] is the best-known book in the area from the 1960s and is still used today. Forsythe and Moler's *Computer Solution of Linear Algebraic Systems* (1967) [2] broke new ground by including listings of computer programs (in several languages) for solving linear systems by Gaussian elimination. In *Solving Least Squares Problems* (1974) [6], Lawson and Hanson gave the first comprehensive treatment of linear least squares problems. The most successful early general textbook was Stewart's *Introduction to Matrix Computations* (1973) [8], which was notable for giving a thorough and insightful treatment of the QR algorithm for eigenproblems. An influential monograph on the symmetric eigenproblem is Parlett's *The Symmetric Eigenvalue Problem* (1980) [7]. The current "bible" of numerical linear algebra, Golub and Van Loan's *Matrix Computations* (1996) [3], is now in its third edition, having first been published in 1983. All these books have given me great enjoyment. I learned numerical linear algebra from Stewart's book as an undergraduate, spent a summer reading Parlett prior to beginning graduate work, and, as an M.Sc. student, read one of the first copies of Golub and Van Loan to reach the UK. These and other books have set a high standard of exposition for later authors in the area to live up to.

Since the early 1980s books in numerical linear algebra have been published at a growing pace. For this review I have chosen three of the most recent textbooks: Datta (1995), Demmel (1997), and Trefethen and Bau (1997). Notable earlier textbooks include Hager (1988) [4], Strang (1988) [10], and Watkins (1991) [11]; my own contribution is [5]; and the first volume of Stewart's *Matrix Algorithms* [9] appeared as this review was being written. Datta is aimed at undergraduate and beginning graduate level courses, while Demmel and Trefethen and Bau target graduate level courses. All three books have been extensively classroom tested. All give MATLAB exercises, and Datta and Demmel provide MATLAB software that is used in the exercises and is available over the Web (from ftp://ftp.mathworks.com/pub/books/datta/ and http://www.siam.org/books/demmel/demmel_class, respectively).

All the books cover linear systems (solution by direct and iterative methods), the least squares problem, eigenvalue and singular value problems, and aspects of numer-

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Structure of Dynamical Systems: A Symplectic View of Physics. *By Jean-Marie Souriau.* Translated from the French by C. H. Cushman-de Vries. Birkhäuser, Boston, MA, 1997. $89.50. xxxiv+403 pp., hardcover. ISBN 0-8176-3695-1.

Undoubtedly, many mathematicians and physicists worldwide will welcome this translation into English of the famous Souriau book originally published in French almost three decades ago. This is especially true for those who remember that the announcements of its translation into English and Russian were made so many years ago. Interested readers should give thanks to Birkhäuser; the translator, C. H. Cushman-de Vries; and the translation editors, R. C. Cushman and G. M. Tuynman, for the really superb work they have done in finally providing the English-reading audience with this volume.

The essence of the book is contained in its subtitle, which has been added by the translation editors. They have also made some slight changes in notation (all of them approved by the author) in order to avoid possible confusion with well-established notation in the present-day literature. Otherwise, all efforts were made to keep the English edition as close as possible to the original. This refers even to marginal symbols that include trumpets and astrological signs and that might look somewhat strange to some readers; the reviewer's opinion is that they are just one more sign of the imagination of the author.

From a formal point of view, the book consists of five chapters of approximately equal length: I. Differential Geometry; II. Symplectic Geometry; III. Mechanics; IV. Statistical Mechanics; V. A Method of Quantization. In most places detailed proofs are omitted, but the analytical arguments are presented or a reference is given for them. The inclusion of definitions of such items as homotopy, cohomologies, foliations, Riesz spaces, measures, etc., hints that this is a book that goes far beyond the standard treatment of mechanics. Having given a rough idea of the contents of the book, let us describe also the situation at the time it first appeared. Starting with the works of Hertz [1] and Hamel [2] on axiomatization of mechanics, classical dynamics transmuted unnoticed into a mathematical discipline. The first application of geometry in dynamics was by Ricci and Levi-Civita [3]. The subsequent developments in this field were summarized in the books by Kasner [4] and Synge [5]. It should be mentioned that these considerations were within the realm of Riemannian or Finslerian geometry of the configurational manifold. Roughly speaking, the geometrical models of mechanics presented there can be split into two principal classes: kinematical and dynamical. In the former, the forces acting on the system are included in neither the metric nor the connection, while in the latter, they are included in both. Some systems allow modeling of both kinds; others allow only one. Each of these models has its merits and shortcomings. A parallel development was initiated by Poincaré and Cartan. It is based on integral invariants and exterior differential two-forms of appropriate rank in the phase space. Gallissot's thesis [6] of 1952 presents a clear exposition of this new approach. A more rigorous mathematical treatment of Hamiltonian and Lagrangian formalism based on symplectic geometry was given in the book by Godbillon [7], which appeared just a year before that of Souriau [8]. Let us remark that the above history of mechanics is quite incomplete and is simply intended to orient the reader. The first 11 references listed below and the bibliography supplied by the translation editors provide the missing parts.

Now, what makes the book under review so special in this context? The answer can be traced in several directions.

First, it introduces some new notions such as the space of motions (the orbit manifold) of a dynamical system, the momentum map (a unifying concept for the constants of motion), and an elementary system that turned out to be quite important for the subsequent development of mechanics itself. The situation here is reminiscent of the Klein view on geometry, as Souriau clearly states, in that the type of mechanics is determined by the Lie group of transformations which preserve the symplectic structure.

Second, it provides a mathematical model not only of classical mechanics but of statistical and quantum mechanics as well. For example, "states" in statistical mechanics are formulated as probability measures on the space of motions. Again, groups enter into the picture and one can obtain classical statistical mechanics by consideration of the group of temporal displacements. Substituting this group with either the Galilei or Poincaré group, many new phenomena can be described.

Quantum mechanics is modeled as follows. If possible, a principal circle bundle (a prequantum manifold) over the phase space of the given dynamical system is built. The necessary and sufficient conditions for its existence are stated and applied to concrete physical systems. "State vectors" are identified with the functions on the prequantum manifold, while "quantum observables" are lifted vector fields of the vector fields generated by the classical ones (i.e., the smooth real functions on the phase space). An alternative scheme based on line, instead of principal bundles, was developed simultaneously by Kostant [12], and these constructions are combined under the name *geometric quantization*. When applied to a homogeneous symplectic manifold on which a Lie group G acts transitively, the problem of quantization turns out to be equivalent to that of building the unitary irreducible representations of G. For a long time the analogies between quantum mechanics and representation theory served as inspiration for both disciplines, and the Kostant–Souriau theory in conjunction with the Kirillov [13] method of orbits provided the precise connections.

Third, and most important, a large number of the principles presented are of a conjectural character and have since been justified. Let us say also that the list of published works on geometric quantization prepared by Puta [14] in 1983 occupied 76 pages, while the total number of references to the French edition cannot even be estimated!

Last but not least, nothing has been changed by the author for the new edition, and this can be interpreted as a hint that possibilities for new applications are still open. What else can one want from a book? The challenge is real!

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