



## BOOK REVIEW

*Introduction à la théorie des groupes de Lie réels*, by Dominique Paul Chevallier, Ellipses, Paris, 2006, xii + 370pp., ISBN 978-2-7298-3068-7.

As the author himself explains the origin of this book was the to summarize the groups theory and it was written as an introduction for the third cycle (PhD) course devoted to differential geometry and mechanics in Pierre et Marie Curie University. The Lie groups themselves play an important role in several branches of Mathematics, Physics and modern Mechanics, and it appeared that an appropriate development of these course Notes can lead to a very synthetic introductory book. It seems also that it will be very useful in comparison with other treatises in French language devoted to the subject, which often present difficulties in their reading by the students. This theory is based on several branches of Mathematics, principally the Lie algebra theory, topology, topological groups and the differential calculus on manifolds and it can not be approached at relatively elementary level. However only some basic knowledge of these disciplines are required to start with the study of the subject and for approaching some significant results. Definitely, for reading of the book only a standard knowledge at the level of the second cycle is sufficient, except perhaps certain more subtle issues discussed in its Appendices.

Besides, the idea to entering directly into the subject leads to the choice to write the book by assuming that the basic concepts from Lie algebras theory are known. These concepts come into play however very gradually by starting with the most elementary definitions intervening in the *Chapter II* and, if it is necessary, can in all cases be acquired in the course of reading the brief report in *Appendix A*.

The subjects developed here are associated with two points of view on the theory of the real Lie groups and this corresponds to the two historical streams in the evolution of the main ideas. On one hand the differential calculus on Lie groups, and the evidence that to each Lie group is attached one Lie algebra that describes the infinitesimal aspects and purely local properties.

On the other side and what is related to their global structure is the possibility to

make use of the properties of the Lie algebras and of their sub-algebras for studying those of the groups and their sub-groups along various relations to topology and differential Geometry. The first point of view corresponds also to the needs of the physicists and the mechanists who must above all be adequate to carry out calculations with these groups and to know the techniques of differential calculations related to the structure of the groups or the structure of differentiable manifolds. An objective of the book is to show how the methods of the differential calculus on Lie Group are organized naturally around the key concept of the Maurer-Cartan form, some properties of the adjoint and co-adjoint representations and the exponential function.

The second point of view interests more the mathematicians and various theoretically minded scientists. It requires obviously some tools regarding the structure of the integrals of the differential systems on manifolds and the Frobenius theorem about sub-manifolds and their topology. Its development was limited to the principal points that constitute a base for reading of more specialized books.

Certain types of groups are not treated in the book at all. In particular these are the complex Lie groups and the Lie groups of infinite dimensions. However, if one limits himself just to calculus, their properties are not quite different from those of the real Lie groups.

One can adopt another point of view to present the subject as well. E.g., it is possible that the exposition be based more directly on the theory of the analytical functions and the formal series and the advantages offered by the algebraic methods. This route is definitively rather near to the methods employed by the founders of the theory. These ideas, quickly evoked at the end of Chapter I, are developed in many other books on the subject.

Exercises, of which a certain number constitute real problems, are grouped at the end of the book. They are presented in a order which do not follow exactly the plan of the book but as a whole are strictly attached to the particular chapters.

As a conclusion - the book meets exactly the main idea of the series in which it appears – to present the subject in a simple manner by providing enough details and mathematical rigour.

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