



## DIFFERENTIAL GEOMETRY OF MOVING SURFACES AND ITS RELATION TO SOLITONS

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**Abstract.** In this article we present an introduction in the geometrical theory of motion of curves and surfaces in  $\mathbb{R}^3$ , and its relations with the nonlinear integrable systems. The working frame is the Cartan's theory of moving frames together with Cartan connection. The formalism for the motion of curves is constructed in the Serret-Frenet frames as elements of the bundle of adapted frames. The motion of surfaces is investigated in the Gauss-Weingarten frame. We present the relations between types of motions and nonlinear equations and their soliton solutions.

### 1. Introduction

Realistic models for many-body or collective interactions involve nonlinear dynamics therefore a large part of interesting and intriguing phenomena cannot be explained or predicted by the corresponding linear approximations. Nonlinearity of the dynamics involves, among other things, a weaker type of uniqueness of solutions especially when the solitary waves have compact support (e.g. compactons) or when the configuration space is a compact manifold (e.g. circle, sphere). The most useful nonlinear systems are of course the integrable ones, i.e., those solvable by inverse scattering theory. These particular systems have soliton solutions and infinite number of conservation laws. The traditional nonlinear systems: Korteweg-de Vries, modified Korteweg-de Vries, sine-Gordon, Schrödinger nonlinear equation and Kadomtsev-Petviashvili were investigated in numerous works and books (see for example the following books and the references listed herein [1, 6, 10–12, 17, 22, 41]).

Many of the integrable nonlinear systems have equivalent representations in terms of differential geometry of curves and surfaces in Riemannian spaces. Such geometric realizations provide a deeper insight into the structure of integrable equations, as well as new physical interpretations [19]. That is why the theory of motions of curves and surfaces, which includes vortices, filaments, and interface dynamics, represents an important emerging field for mathematics, engineering and