



VECTOR DECOMPOSITIONS OF ROTATIONS

DANAİL BREZOV, CLEMENTINA MLADENOVA AND IVAĬLO MLADENOV

Presented by Ivaĭlo M. Mladenov

Abstract. Here we derive analytic expressions for the scalar parameters which appear in the generalized *Euler* decomposition of the rotational matrices in \mathbb{R}^3 . The axes of rotations in the decomposition are almost arbitrary and they need only to obey a simple condition to guarantee that the problem is well posed. A special attention is given to the case when the rotation is decomposable using only two rotations and for this case quite elegant expressions for the parameters were derived.

In certain cases one encounters infinite parameters due to the rotations by an angle π (the so called *half turns*). We utilize both geometric and algebraic methods to obtain those conditions that can be used to predict and deal with various configurations of that kind and then, applying *l'Hôpital's* rule, we easily obtain the solutions in terms of linear fractional functions. The results are summarized in two Tables and a flowchart presenting in full details the procedure.

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