



**SINE–GORDON SOLITONS, KINKS AND BREATHERS  
 AS PHYSICAL MODELS OF NONLINEAR EXCITATIONS  
 IN LIVING CELLULAR STRUCTURES**

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**Abstract.** Nonlinear space-time dynamics, defined in terms of celebrated ‘solitonic’ equations, brings indispensable tools for understanding, prediction and control of complex behaviors in both physical and life sciences. In this paper, we review sine–Gordon solitons, kinks and breathers as models of nonlinear excitations in complex systems in physics and in living cellular structures, both intra–cellular (DNA, protein folding and microtubules) and inter–cellular (neural impulses and muscular contractions).

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