



# EHRENFEST THEOREM IN PRECANONICAL QUANTIZATION

IGOR V. KANATCHIKOV

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**Abstract.** We discuss the precanonical quantization of fields which is based on the De Donder–Weyl (DW) Hamiltonian formulation which treats the space and time variables on an equal footing. Classical field equations in DW Hamiltonian form are derived as the equations on the expectation values of precanonical quantum operators. This field-theoretic generalization of the Ehrenfest theorem demonstrates the consistency of three aspects of precanonical field quantization: (i) the precanonical representation of operators in terms of the Clifford (Dirac) algebra valued partial differential operators, (ii) the Dirac-like precanonical generalization of the Schrödinger equation without the distinguished time dimension, and (iii) the definition of the scalar product in order to calculate expectation values of operators using the precanonical wave functions.

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*Keywords:* quantum field theory, precanonical quantization, De Donder-Weyl theory, Clifford algebra, Yang-Mills theory, curved space-time, Schrödinger functional, polysymplectic structure.

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