JGSP **39** (2015) 17–31

JOURNAL OF
Geometry and Symmetry in Physics
ISSN 1312-5192

ON MKDV EQUATIONS RELATED TO THE AFFINE KAC-MOODY ALGEBRA $A_5^{(2)}$

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Communicated by Boris Konopeltchenko

Abstract. We have derived a new system of mKdV-type equations which can be related to the affine Lie algebra $A_5^{(2)}$. This system of partial differential equations is integrable via the inverse scattering method. It admits a Hamiltonian formulation and the corresponding Hamiltonian is also given. The Riemann-Hilbert problem for the Lax operator is formulated and its spectral properties are discussed.

MSC: 22E46, 53C35, 57S20
Keywords: Affine Kac-Moody algebras, modified KdV equations, Riemann-Hilbert problems

1. Introduction

The general theory of the nonlinear evolution equations (NLEE) allowing Lax representation is well developed [1, 3, 6, 9, 10, 21]. In this paper our aim is to derive a set of modified Korteveg–de Vries (mKdV) equations related to three affine Lie algebras using the procedure introduced by Mikhailov [20]. This means that the equations can be written as the commutativity condition of two ordinary differential operators of the type

$$\begin{aligned} L\psi &\equiv \mathrm{i}\frac{\partial\psi}{\partial x} + U(x,t,\lambda)\psi = 0 \\ M\psi &\equiv \mathrm{i}\frac{\partial\psi}{\partial t} + V(x,t,\lambda)\psi = \psi\Gamma(\lambda) \end{aligned} \tag{1}$$

where $U(x,t,\lambda)$, $V(x,t,\lambda)$ and $\Gamma(\lambda)$ are some polynomials of λ to be defined below. We request also that the Lax pair (1) possesses appropriate reduction group [20], for example if the reduction group is $\mathbb{Z}_h$ (h is a positive number) the reduction condition is

$$C\big(U(x,t,\lambda)\big) = U(x,t,\omega\lambda), \qquad C\big(V(x,t,\lambda)\big) = V(x,t,\omega\lambda). \tag{2}$$

doi: 10.7546/jgsp-39-2015-17-31