



RICCI FLOW ON MODIFIED RIEMANN EXTENSIONS

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Abstract. We study the properties of the modified Riemann extensions evolving under the Ricci flow with examples.

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1. Introduction

The Ricci flow and the evolution equations of the Riemannian curvature tensor were initially introduced by Hamilton [8] and was later studied to a large extent by Perelman [13–15], Cao and Zhu [4], Morgan and Tian [10]. Indeed, the theory of Ricci flow has been used to prove the geometrization and Poincaré conjectures [1]. However not much work has been done on Ricci flows on modified Riemann extensions. The Ricci flow equation is the evolution equation $\frac{\partial g_{ij}}{\partial t} = -2R_{ij}$ where g_{ij} and R_{ij} are metric components respectively. As flow progresses the metric changes and hence the properties related to it.

Patterson and Walker [11] have defined Riemann extensions and showed how a Riemannian structure can be given to the $2n$ dimensional tangent bundle of an n -dimensional manifold with given non-Riemannian structure. This shows that Riemann extension provides a solution of the general problem of embedding a manifold M carrying a given structure in a manifold \tilde{M} carrying another structure, the embedding being carried out in such a way that the structure on \tilde{M} induces in a natural way the given structure on M . The Riemann extension of Riemannian or non-Riemannian spaces can be constructed with the help of the Christoffel coefficients Γ^i_{jk} of corresponding Riemann space or with connection coefficients Π^i_{jk} in the case of the space of affine connection [5]. The theory of Riemann extensions has been extensively studied by Afifi [1]. Though the Riemann extensions itself is rich in geometry, here in our discussions, the modified Riemann extensions fit naturally in to the frame work. Modified Riemann extensions are introduced in [2] and their properties we list briefly in the next section.