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EXACT INTEGRATION OF A NONLINEAR MODEL OF STEADY HEAT CONDUCTION/RADIATION IN A WIRE WITH INTERNAL POWER

GIOVANNI M. SCARPELLO, ARSEN PALESTINI AND DANIELE RITELLI

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Abstract. The paper treats in one dimensional mixed heat transfer problem of steady conduction and radiation in a wire with internal source. We are led to a Cauchy problem consisting of a second order nonlinear ordinary differential equation. A special integrable case with two non independent left boundary conditions requires a hyperelliptic integral, for which a representation theorem has been established through the Gauss hypergeometric function ${}_2F_1$. The relevant steady solution is then found to grow monotonically with the distance from boundary, up to a certain limiting position where it suddenly jumps unbounded.

1. Introduction

Conduction, namely the flow of thermal energy through solid bodies, was modelled by Jean B. Fourier (1768-1830) who first inquired into the general principles of it. Throughout his *Théorie analytique de la chaleur* (1822), he established a partial differential equation (PDE) for analyzing the temperature distribution within a conducting body. His analytical conduction theory disregards the molecular structure of a body and thinks of it as a continuum, but after Fourier it has been understood that-on the contrary- conduction is actually caused by particle collisions. His *linear* PDE, in one dimensional geometry, is

$$\rho\, c_p \frac{\partial T}{\partial t}(t,x) = \chi\, \frac{\partial^2 T}{\partial x^2}(t,x)$$

where the material data are: thermal conductivity χ, specific heat capacity c_p and volumetric density ρ. As far as it concerns the spatial effects, the PDE has to be solved with suitable *boundary conditions* (BC).

Transient problems $(\partial T/\partial t \neq 0)$ will also need *initial conditions* (IC) on T for every position in the system: the PDE is parabolic and heat propagates at *infinite*