



INVERSION OF DOUBLE-COVERING MAP $\text{SPIN}(N) \rightarrow \text{SO}(N, \mathbb{R})$ FOR $N \leq 6$

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Abstract. This work provides an algorithmic procedure for finding the pair of elements in the spin group which map to a given matrix in the special orthogonal group of order five or six. This is achieved by first solving the problem when the special orthogonal matrix is a Givens rotation, and then exploiting the fact that the covering maps are group homomorphisms and that any special orthogonal matrix can be explicitly decomposed into a product of Givens rotations. For this purpose systems of quadratic equations in several variables have to be solved symbolically. The resulting solution display a transparent dependency on the entries of the Givens matrices.

MSC: 15A66, 15A16

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