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TOPOLOGICAL QUANTUM NUMBERS OF DYONIC FIELDS OVER TAUB-NUT AND TAUB-BOLT SPACES

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Abstract. We calculate the Chern numbers of $\mathrm{SU}(2)$-homogeneous Einstein-Maxwell gravitational instantons with boundary at infinity. By restating these numbers as Chern-Simons invariants on the boundary apparent conflicting results emerge. We resolve this issue examining the topological stability of the self-gravitating Abelian fields. No quantization carrying physical meaning is found when the background is a Taub-NUT space. However the magnetic charge of dyons on Taub-Bolt spaces is found to be of topological quantum nature. In this framework electric charge is quantized by a consistency condition.

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Keywords: Dyons, instantons, Taub-Bolt, topological charge

1. Introduction

The study of gravitational instantons in Euclidean Einstein gravity was initiated in the 70's [6,16,17,24,25,31] to apply the path-integral approach in the hope to construct a theory of quantum gravity. This expectation was based upon the success reached by this approach in the application of quantum field theory to Yang-Mills theories. Indeed, the instanton solutions of classical Euclidean Yang-Mills field equations, with self-dual field strength, allow one to interpret the physical vacuum state as a superposition of an infinite number of vacuum states. This leads to an alternative non-perturbative quantization of Yang-Mills theories by using the path-integral approach. The idea of applying similar analysis to Euclidean Einstein gravity found several conceptual and technical difficulties [11], which are currently under investigation to formulate a consistent theory of quantum gravity. Nevertheless, gravitational instantons became a topic of intensive research, and together with monopoles and solitons constitute the area of theoretical physics which today is known as topological defects.

A different approach was recently proposed in which concepts of topological quantization are applied to find quantum information from classical fields, including