



## SYMMETRY GROUPS OF SYSTEMS OF ENTANGLED PARTICLES

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**Abstract.** A Lorentz transformation of signature  $(m, n)$ ,  $m, n \in \mathbb{N}$ , is a pseudo-rotation in a pseudo-Euclidean space of signature  $(m, n)$ . Accordingly, the Lorentz transformation of signature  $(1, 3)$  is the common Lorentz transformation of special relativity theory. It is known that entangled particles involve Lorentz symmetry violation. Hence, the aim of this article is to expose and illustrate the symmetry groups of systems of entangled particles uncovered in [44]. It turns out that the Lorentz transformations of signature  $(m, n)$  form the symmetry group by which systems of  $m$   $n$ -dimensional entangled particles can be understood, just as the common Lorentz group of signature  $(1, 3)$  forms the symmetry group by which Einstein's special theory of relativity can be understood. Consequently, it is useful to extend special relativity theory by incorporating Lorentz transformation groups of signature  $(m, 3)$  for all  $m \geq 2$ . The resulting extended special relativity theory, then, provides not only the symmetry group of the  $(1 + 3)$ -dimensional spacetime of particles, but also the symmetry group of the  $(m + 3)$ -dimensional spacetime of systems of  $m$  entangled three-dimensional particles, for each  $m \geq 2$ .

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