



EXAMPLES OF AUTOMORPHISM GROUPS OF IND-VARIETIES OF GENERALIZED FLAGS*

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Communicated by Ivařilo M. Mladenov

Abstract. We compute the automorphism groups of finite and cofinite ind-grassmannians, as well as of the ind-variety of the maximal flags indexed by $\mathbb{Z}_{>0}$. We pay special attention to differences with the case of ordinary flag varieties.

MSC: 14L30, 14M15, 14M17

Keywords: Automorphism group, flag ind-variety, ind-grassmannian

1. Introduction

The flag varieties of the classical Lie groups are central objects of study both in geometry and representation theory. In a sense, they are a hub for many directions of research in both fields. Several different infinite-dimensional analogues of the ordinary flag varieties have been studied in the literature, one such analogue being the ind-varieties of generalized flags introduced in [1] and further investigated in [2–5]; see also the survey [6]. The latter ind-varieties are direct limits of classical flag varieties and are homogeneous ind-spaces for the simple ind-groups $SL(\infty)$, $SO(\infty)$, $Sp(\infty)$. Without doubt, some of these ind-varieties, in particular the ind-grassmannians, have been known long before the paper [1].

A natural question of obvious importance is the question of finding the automorphism groups of the ind-varieties of generalized flags. The purpose of the present paper is to initiate a discussion in this direction and to point out some differences with the case of ordinary flag varieties - see Section 4.

2. Automorphisms of Finite and Cofinite Ind-Grassmannians

The base field is \mathbb{C} . Let V be a fixed countable-dimensional complex vector space. We fix a basis $E = \{e_1, \dots, e_n, \dots\}$ of V and set $V_n := \text{span}_{\mathbb{C}}\{e_1, \dots, e_n\}$. Then $V = \cup_n V_n$. Fix $k \in \mathbb{Z}_{>0}$. By definition, $\text{Gr}(k, V)$ is the set of all k -dimensional

*Dedicated to the memory of Professor Vasil V. Tsanov 1948-2017.