



RELATIONS BETWEEN LAPLACE SPECTRA AND GEOMETRIC QUANTIZATION OF RIEMANNIAN SYMMETRIC SPACES*

DIMITAR GRANTCHAROV AND GUEO GRANTCHAROV

Communicated by Ivaïlo M. Mladenov

Abstract. We consider a modified Kostant-Souriau geometric quantization scheme due to Czyz and Hess for Hamiltonian systems on the cotangent bundles of compact rank-one Riemannian symmetric spaces (CROSS). It is used, together with a symplectic reduction process, to relate its energy spectrum to the spectrum of the Laplace-Beltrami operator. Moreover, the corresponding eigenspaces have real dimension equal to the complex dimension of the space of the holomorphic sections of the quantum bundle which is obtained after the quantization. The relation between the two constructions was first noticed by Mladenov and Tsanov for the case of the spheres. In addition to the CROSS case, we announce preliminary results related to the case of compact Riemannian symmetric spaces of higher rank.

MSC: 32M10, 53C35, 58D50

Keywords: Geodesic flow, Riemannian symmetric spaces, quantization

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*Dedicated to the memory of Professor Vasil V. Tsanov 1948-2017.