



ON THE GEOMETRY OF ORBITS OF CONFORMAL VECTOR FIELDS

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Abstract. Geometry of orbit is a subject of many investigations because it has important role in many branches of mathematics such as dynamical systems, control theory. In this paper it is studied geometry of orbits of conformal vector fields. It is shown that orbits of conformal vector fields are integral submanifolds of completely integrable distributions. Also for Euclidean space it is proven that if all orbits have the same dimension they are closed subsets.

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1. Introduction

The integral curves of vector fields and the orbits of an arbitrary set of vector fields on the smooth manifold have been studied in many investigations because of their importance in Control Theory, Dynamical systems, Foliation Theory and Physics [1, 4, 10, 14–16]. To the study of systems of vector fields from the point of view of Control Theory (the properties of accessibility and complete controllability) have been devoted numerous investigations [15, 16]. One of important class of vector fields is class of conformal vector fields which has wide applications. Geometry of conformal vector fields is subject of many papers [2–4, 12]. In this paper we study some properties of orbits of conformal vector fields.

In the paper, smoothness is understood as smoothness of the class C^∞ .

2. Orbits of Vector Fields and Distributions

Let (M, g) be a smooth Riemannian manifold of dimension n with metric tensor g , $V(M)$ – a set of all smooth vector fields on a manifold M . The set $V(M)$ is a