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SECANT VARIETIES AND DEGREES OF INVARIANTS*

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Abstract. The ring of invariant polynomials $\mathbb{C}[V]^G$ over a given finite dimensional representation space V of a connected complex reductive group G is known, by a famous theorem of Hilbert, to be finitely generated. The general proof being nonconstructive, the generators and their degrees have remained a subject of interest. In this article we determine certain divisors of the degrees of the generators. Also, for irreducible representations, we provide lower bounds for the degrees, determined by the geometric properties of the unique closed projective G-orbit $\mathbb{X}$, and more specifically its secant varieties. For a particular class of representations, where the secant varieties are especially well behaved, we exhibit an exact correspondence between the generating invariants and the secant varieties intersecting the semistable locus.

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1. Introduction

Let G be a connected complex reductive algebraic group and V be a finite dimensional G-module. A famous theorem of Hilbert asserts that the ring of invariant polynomials $\mathbb{C}[V]^G$ is finitely generated. The general proof being nonconstructive, the generators and their degrees have remained a topic of significant interest. The generators can be chosen homogeneous. The finite set of generators is clearly not unique, but, their degrees of a minimal set of generators are uniquely determined, if one convenes to an increasing order and takes multiplicity into account. We say that $\mathbb{C}[V]^G$ admits a generator of degree $d > 0$, if the degree component $\mathbb{C}[V]^G_d$ is not contained in the subring generated by $\mathbb{C}[V]^G_{<d}$. The maximal degree of a generator is called the Noether number, $\mathrm{No}(G, V)$, this is the minimal d for which $\mathbb{C}[V]^G_{\leq d}$ generates $\mathbb{C}[V]^G$. If $\mathbb{C}[V]^G \neq \mathbb{C}$, we denote by d_1 the minimal positive degree of a generator.

*To the memory of my father - Vasil V. Tsanov 1948-2017.

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