



ON Λ -ELASTICA

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Communicated by Ivailo M. Mladenov

Abstract. In this paper, we investigate a transition from an elastica to a piece-wise elastica whose connected point defines the hinge angle ϕ_0 and we call the piece-wise elastica Λ_{ϕ_0} -*elastica* or Λ -*elastica*. Such transition appears in the bending beam experiment when an elastic beam is gradually compressed and at some moment suddenly due to the rupture, the shapes of Λ -elastica appear. We construct a mathematical theory to describe the phenomena and represent the Λ -elastica in terms of the elliptic ζ -function completely. Using the mathematical theory, we discuss the experimental results from an energetic viewpoint and numerically show the explicit shape of Λ -elastica. It means that this paper provides a novel investigation on elastica theory with rupture.

MSC: 74B99, 74R99, 74K10, 33E05

Keywords: Elastica, hinge angle, Λ -elastica, rupture

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	doi: 10.7546/jgsp-54-2019-13-35	13