ON GIBBS STATES OF MECHANICAL SYSTEMS WITH SYMMETRIES∗

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Communicated by Ivaïlo M. Mladenov

Abstract. The French mathematician and physicist Jean-Marie Souriau studied Gibbs states for the Hamiltonian action of a Lie group on a symplectic manifold and considered their possible applications in Physics and Cosmology. These Gibbs states are presented here with detailed proofs of all the stated results. A companion paper to appear will present examples of Gibbs states on various symplectic manifolds on which a Lie group of symmetries acts by a Hamiltonian action, including the Poincaré disk and the Poincaré half-plane.

MSC: 53D05, 53D20, 53D17, 82B03, 82B30

Keywords: Gibbs states, Hamiltonian systems, Liouville measure, moment maps, symplectic and Poisson manifolds, thermodynamic equilibrium

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∗In memory of the French mathematician and physicist Jean-Marie Souriau (1922–2012).