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QUANTUM HALL EFFECT AND NONCOMMUTATIVE GEOMETRY

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Abstract. We study magnetic Schrödinger operators with random or almost periodic electric potentials on the hyperbolic plane, motivated by the quantum Hall effect (QHE) in which the hyperbolic geometry provides an effective Hamiltonian. In addition we add some refinements to earlier results. We derive an analogue of the Connes-Kubo formula for the Hall conductance via the quantum adiabatic theorem, identifying it as a geometric invariant associated to an algebra of observables that turns out to be a crossed product algebra. We modify the Fredholm modules defined in [4] in order to prove the integrality of the Hall conductance in this case.

1. Introduction

In [4], continuous and discrete magnetic Hamiltonians containing terms arising from a background hyperbolic geometry were introduced. These may be thought of as effective Hamiltonians for an analogue of the quantum Hall effect studied in a Euclidean model by Bellissard [2] and Xia [20]. We interpret these Hamiltonians, following a suggestion of Bellissard, as modelling spinless electrons in a conducting material with a perturbation term arising from a background hyperbolic geometry. (In [4] we took the somewhat different view that the conducting material exhibited hyperbolic geometry.) They motivate constructing Fredholm modules associated in a natural way with Riemann surfaces and two dimensional orbifolds which give a higher genus analogue of the work of Bellissard (which is the genus one case) on the quantum Hall effect. In [4] we considered Hamiltonians invariant under a projective action of a Fuchsian group Γ. We will only discuss groups whose actions on hyperbolic space are free here and refer the reader to [12] for the more general case. In this paper we allow in addition a random potential (which may be thought of as modelling impurities) so that the invariance of the