



## CLOSED GEODESICS ON CERTAIN SURFACES OF REVOLUTION

JAMES C. ALEXANDER

Communicated by John Oprea

---

**Abstract.** Recently I. Mladenov and J. Oprea have investigated a number of surfaces of revolution, and in particular, developed numerical shooting methods to investigate geodesics on those surfaces, which in turn led them to raise some questions concerning closed geodesics on those surfaces. Here we develop explicit formulae, usually in terms of elliptic integrals, that permit us to answer the questions. The computations are based of course on the classical Clairaut's formulae, and a major point is to demonstrate that explicit computations can be made. A closed geodesic on a surface of revolution oscillates  $q$  times across the equator of the surface, while winding  $p$  times around the axis of rotation. Call the pair  $(p, q)$  the *type* of the geodesic. In particular, the permitted types are explicitly determined for the investigated surfaces.

---

### 1. Introduction

Recently I. Mladenov and J. Oprea have considered some particular surfaces of revolution [8–10], characterizing them via variational principles, and also considered geodesics on them. They developed Maple code for visualization and other computation. In particular, they coded a shooting method to construct closed geodesics. Motivated by this work, especially of [9], our purpose here is to demonstrate how explicit calculations can be done. For many surfaces, including those investigated by Mladenov and Oprea [8,9], we develop explicit formulas, in terms of complete elliptic integrals. For others, including the Hopf surfaces in [10], the treatment is slightly different.

The surfaces we consider are rotated around the  $z$ -axis and are symmetric with respect to the  $xy$ -plane (although this last is not necessary). The 'equator' in the  $xy$ -plane is a geodesic. Other geodesics intersect the equator. A closed geodesic winds around the  $z$ -axis  $p$  times while making  $q$  oscillations across the equator (hence  $2q$  intersections). We explicitly determine the permitted  $p$  and  $q$ , and in