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COHERENT STATES ASSOCIATED TO THE JACOBI GROUP - A VARIATION ON A THEME BY ERICH KÄHLER

STEFAN BERCEANU

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Abstract. Using the coherent states attached to the complex Jacobi group – the semi-direct product of the Heisenberg-Weyl group with the real symplectic group – we study some of the properties of coherent states based on the manifold which is the product of the n-dimensional complex plane with the Siegel upper half plane.

1. Introduction

In this paper we continue the investigation of the Jacobi group [7,8,16] – the semi-direct product of the Heisenberg-Weyl group and the symplectic group – started in [4,5], using Perelomov's coherent states (CS). The Jacobi group is an important object in connection with Quantum Mechanics, Geometric Quantization, Optics, etc., [2,9,14,15,18,20].

Applying the methods developed in [3], in [4] we have constructed generalized CS attached to the Jacobi group $G_1^J = H_1 \rtimes \mathrm{SU}(1,1)$, based on the homogeneous Kähler manifold $\mathcal{D}_1^J = H_1/\mathbb{R} \times \mathrm{SU}(1,1)/\mathrm{U}(1) = \mathbb{C}^1 \times \mathcal{D}_1$. Here $\mathcal{D}_1$ denotes the unit disk $\mathcal{D}_1 = \{w \in \mathbb{C};\, |w| < 1\}$, and H_n is the $(2n+1)$-dimensional real Heisenberg-Weyl group with Lie algebra $\mathfrak{h}_n$. In [4] we have also emphasized that, when expressed in appropriate coordinates on the manifold $\mathcal{X}_1^J = \mathbb{C} \times \mathcal{H}_1$, $\mathcal{H}_1 = \{v \in \mathbb{C};\, \mathrm{Im}(v) > 0\}$, the Kähler two-form ω_1 is identical with the one considered by Kähler-Berndt [6,7,10–12].

In [5] we have considered coherent states attached to the Jacobi group $G_n^J = H_n \rtimes \mathrm{Sp}(n,\mathbb{R})$, based on the manifold $\mathcal{D}_n^J = \mathbb{C}^n \times \mathcal{D}_n$, where $\mathcal{D}_n$ is the Siegel ball $\mathcal{D}_n = \{W \in M(n,\mathbb{C});\, W = W^t, 1 - W\bar{W} > 0\}$. In this paper we calculate the Kähler two-form ω'_n on the manifold $\mathcal{X}_n^J = \mathbb{C}^n \times \mathcal{H}_n$, where $\mathcal{H}_n$ is the Siegel upper half plane obtained by the Cayley transform of the Siegel ball $\mathcal{D}_n$. This ω'_n is a "n"-dimensional generalization of Kähler-Berndt's two-form ω'_1 on $\mathcal{X}_1^J$ to the corresponding one on $\mathcal{X}_n^J$. The physical relevance of these results follows from