



## ACCARDI COMPLEMENTARITY IN $\mu$ -DEFORMED QUANTUM MECHANICS

CLAUDIO PITA and STEPHEN BRUCE SONTZ

Communicated by Martin Schlichenmaier

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**Abstract.** In this note we show that the momentum and position operators of  $\mu$ -deformed quantum mechanics for  $\mu > 0$  are not Accardi complementary in a sense that we will define. We conjecture that this is also true if  $-1/2 < \mu < 0$ .

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### 1. Introduction

We begin by reviewing some basics of  $\mu$ -deformed quantum mechanics. This comes from Rosenblum [8]. For recent related work see references [2], [6] and [7]. We consider a deformation of quantum mechanics depending on a parameter  $\mu > -1/2$ , which will be fixed throughout this discussion. We work in the complex Hilbert space  $L^2(\mathbb{R}, m_\mu)$ , where the measure  $m_\mu$  for  $x \in \mathbb{R}$  (the real line) is given by  $dm_\mu(x) := [2^{\mu+1/2}\Gamma(\mu+1/2)]^{-1}|x|^{2\mu}dx$ . Here  $dx$  is Lebesgue measure on  $\mathbb{R}$  and  $\Gamma$  is the Euler gamma function. (The normalization constant will be explained later.) In this Hilbert space we have two unbounded self-adjoint operators:  $Q_\mu$ , the  $\mu$ -deformed position operator, and  $P_\mu$ , the  $\mu$ -deformed momentum operator. They are defined for  $x \in \mathbb{R}$  and certain elements  $\psi \in L^2(\mathbb{R}, m_\mu)$  by

$$Q_\mu\psi(x) := x\psi(x)$$

$$P_\mu\psi(x) := \frac{1}{i} \left( \psi'(x) + \frac{2\mu}{x}(\psi(x) - \psi(-x)) \right).$$

We omit details about exact domains of definition. Interest in these operators originates in Wigner [9] where equivalent forms of them are used as examples of operators that do not satisfy the usual canonical commutation relation in spite of the fact that they do satisfy the equations of motion  $[H_\mu, Q_\mu] = P_\mu$  and  $[H_\mu, P_\mu] = -Q_\mu$  for the Hamiltonian  $H_\mu := \frac{1}{2}(Q_\mu^2 + P_\mu^2)$ . What does hold is the  $\mu$ -deformed canonical commutation relation:  $i[P_\mu, Q_\mu] = I + 2\mu J$ , where  $I$  is the identity operator and  $J$  is the parity operator  $J\psi(x) := \psi(-x)$ .