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A HOLOMORPHIC REPRESENTATION OF THE SEMIDIRECT SUM OF SYMPLECTIC AND HEISENBERG LIE ALGEBRAS

STEFAN BERCEANU

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Abstract. A representation of the Jacobi algebra by first order differential operators with polynomial coefficients on a Kähler manifold which as set is the product of the complex multidimensional plane times the Siegel ball is presented.

1. Introduction

In this paper we construct a holomorphic polynomial first order differential representation of the Lie algebra which is the semidirect sum $\mathfrak{h}_n \rtimes \mathfrak{sp}(2n, \mathbb{R})$, on the manifold $\mathbb{C}^n \times \mathcal{D}_n$, different from the extended metaplectic representation [6]. The case $n = 1$ corresponding to the Lie algebra $\mathfrak{h}_1 \rtimes \mathfrak{su}(1,1)$ was considered in [3]. The natural framework of such an approach is furnished by the so called coherent state (CS)-groups, and the semi-direct product of the Heisenberg-Weyl group with the symplectic group is an important example of a mixed group of this type [11]. We use Perelomov's coherent state aproach [12]. Previous results concern the hermitian symmetric spaces [2] and semisimple Lie groups which admit CS-orbits [4]. The case of the symplectic group was previously investigated in [1], [6], [5], [10], [12]. Due to lack of space we do not give here the proofs, but in general the technique is the same as in [3], where also more references are given. More details and the connection of the present results with the squeezed states [13] will be discussed elsewhere.

2. The Differential Action of the Jacobi Algebra

The Heisenberg-Weyl (HW) group is the nilpotent group with the $2n+1$-dimensional real Lie algebra $\mathfrak{h}_n =< \mathrm{i}s1 + \sum_{i=1}^{n}(x_i a_i^+ - \bar{x}_i a_i) >_{s\in\mathbb{R}, x_i\in\mathbb{C}}$, where a_i^+ (a_i) are the boson creation (respectively, annihilation) operators.