

## ON THE GEOMETRY OF BIHARMONIC SUBMANIFOLDS IN SASAKIAN SPACE FORMS\*

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**Abstract.** We classify all proper-biharmonic Legendre curves in a Sasakian space form and point out some of their geometric properties. Then we provide a method for constructing anti-invariant proper-biharmonic submanifolds in Sasakian space forms. Finally, using the Boothby-Wang fibration, we determine all proper-biharmonic Hopf cylinders over homogeneous real hypersurfaces in complex projective spaces.

### 1. Introduction

As defined by Eells and Sampson in [14], **harmonic maps**  $f : (M, g) \rightarrow (N, h)$  are the critical points of the **energy functional**

$$E(f) = \frac{1}{2} \int_M \|df\|^2 v_g$$

and they are solutions of the associated Euler-Lagrange equation

$$\tau(f) = \text{trace}_g \nabla df = 0$$

where  $\tau(f)$  is called the **tension field** of  $f$ . When  $f$  is an isometric immersion with mean curvature vector field  $H$ , then  $\tau(f) = mH$  and  $f$  is harmonic if and only if it is minimal.

The **bienergy functional** (proposed also by Eells and Sampson in 1964, [14]) is defined by

$$E_2(f) = \frac{1}{2} \int_M \|\tau(f)\|^2 v_g.$$

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