Tenth International Conference on
Geometry, Integrability and Quantization
June 6–11, 2008, Varna, Bulgaria
Ivaïlo M. Mladenov, Gaetano Vilasi
and Akira Yoshioka, Editors
Avangard Prima, Sofia 2009, pp 237–247

NONCOMMUTATIVE DEFORMATION OF INSTANTONS AND VORTEXES*

AKIFUMI SAKO

*Department of General Education, Kushiro National College of Technology
Otanoshike-Nishi 2-32-1, Kushiro 084-0916, Japan*

Abstract. We study noncommutative (NC) instantons and vortexes. At first, we construct instanton solutions which are deformations of instanton solutions on commutative Euclidean four-space. We show that the instanton numbers of these NC instanton solutions coincide with the commutative solutions. Next, we also deform vortex solutions similarly and we show that their vortex numbers are unchanged under the NC deformation.

1. Introduction

Instanton connections in the four dimensional Yang-Mills theory are defined by

$$F^{+} = \frac{1}{2}(1+*)F = 0 \tag{1}$$

where F is a curvature two-form and $*$ is the Hodge star operator.

The NC instanton solutions were constructed with the ADHM method in [1, 15]. The ADHM construction which generate the instanton $\mathrm{U}(N)$ gauge field require a pair of the two complex vector spaces $V = \mathbb{C}^k$, $W = \mathbb{C}^N$. Here k is an integer. Introduce B_1, $B_2 \in \mathrm{Hom}(V,V)$, $I \in \mathrm{Hom}(W,V)$ and $J \in \mathrm{Hom}(V,W)$ called ADHM data such that

$$\mu_{\mathbf{R}} := [B_1, B_1^\dagger] + [B_2, B_2^\dagger] + II^\dagger - J^\dagger J = \zeta\, \mathrm{Id}_k \tag{2}$$

$$\mu_{\mathbf{C}} := [B_1, B_2] + IJ = 0. \tag{3}$$

Here ζ is a NC parameter and its detail will appear in the following. Using these ADHM data we can construct NC instanton and call it NC ADHM instanton in the following. NC ADHM instantons and deformed instantons from the commutative ADHM construction is unknown.

---

*Reprinted from JGSP **14** (2009) 85–96.